18th Orthogonal Polynomials, Special Functions and Applications – OPSFA-18
17–21 Aug 2026 | Doshisha University, Kyoto, Japan

Generated: 2026-08-07 12:49:11+09:00

OPSFA-18 Kyoto – 18th Orthogonal Polynomials, Special Functions and Applications

Time	Day 1	Day 2	Day 3	Day 4	Day 5
9:00 | 10:00		**Makiko Sasada**	**Benjamin Eichinger**	**María Ángeles García-Ferrero**	**Dan Dai**
10:30 | 12:00		TS 01, TS 09, TS 13, TS 07, TS 14	TS 01, TS 09, TS 11, TS 05, TS 16	TS 02, TS 10, TS 12, TS 05, TS 15	TS 02, TS 08, TS 04, TS 06, TS 15
12:00 | 13:30	**Reception Desk Opens** (12:00)	Lunch	**Poster & Lunch** (12:00-14:30)	Lunch	Lunch
13:30 | 14:30	**Opening** (14:00-14:30)	**Kohei Iwaki**		**Manuela Girotti**	**Rinat Kedem**
15:00 | 16:30	**Jan Felipe van Diejen** (14:30-15:30) **Yasuhiko Yamada** (15:45-16:45)	TS 01, TS 09, TS 13, TS 07, TS 16	**Discussion**	TS 02, TS 08, TS 12, TS 05, TS 15	TS 02, TS 08, TS 04, TS 06, TS 15
16:45 | 18:15	**Guilherme Silva** (17:00-18:00)	TS 01, TS 09, TS 13 / TS 03, TS 07, TS 16	**Discussion**	TS 02, TS 08, TS 12, TS 05, TS 15	**Closing** (16:30–17:00)
18:00 | 20:00	Welcome Party (18:00–20:00) @ Hamac de Paradis (1F)	SIAG/OPSF and OPSFA Business Meeting (18:15-19:00)	Banquet (18:30–20:30) @ Hotel Okura Kyoto		

List of Thematic Sessions

- **TS 01** Painlevé and Integrable Systems (11)
- **TS 03** Exact WKB Analysis and Related Topics (Topological Recursion, Conformal Blocks, Beta-Ensembles) (2)
- **TS 05** Algebraic Combinatorics (11)
- **TS 07** Quantum Walks and Quantum Information (8)
- **TS 09** Multiple Orthogonal Polynomials (12)
- **TS 11** Orthogonal Polynomials on the Unit Circle (2)
- **TS 13** Multivariate Special Functions (7)
- **TS 15** General Orthogonal Polynomials, Hypergeometric and Special Functions (including Mathieu, Lamé, Heun, etc.) (14)
- **TS 02** Random Matrices and Probability (15)
- **TS 04** Orthogonal Polynomials, Mathematical Physics, and Interacting Particle Systems (5)
- **TS 06** Combinatorics, q-Series, Partitions, and Number Theory (4)
- **TS 08** Asymptotics of Orthogonal Polynomials and Special Functions (11)
- **TS 10** Matrix Orthogonal Polynomials (3)
- **TS 12** Exceptional Orthogonal Polynomials (8)
- **TS 14** Representation Theory and Special Functions (3)
- **TS 16** Applications of Orthogonal Polynomials and Special Functions – Interdisciplinary Topics (including Numerical Analysis, Data Science, Machine Learning, etc.) (8)

Day 1 (August 17th)

Day 1	Hardy Hall
14:00 | 14:30	Opening
Chair	Luc Vinet
14:30 | 15:30	**Jan Felipe van Diejen:** Spectrum of quantum relativistic Toda molecule via Hall-Littlewood polynomials
15:30 | 15:45	Break
Chair	Teruhisa Tsuda
15:45 | 16:45	**Yasuhiko Yamada:** Quantum discrete isomonodromic deformations
16:45 | 17:00	Break
Chair	Inés Pacharoni
17:00 | 18:00	**Guilherme Silva:** Orthogonal polynomials at finite temperature
18:00 | 20:00	Welcome Party@Hamac de Paradis (1F)

Day 2 (August 18th)

Day 2	Hardy Hall (Chair: Satoshi Tsujimoto)				
9:00 – 10:00	**Makiko Sasada:** Stationary distributions and macroscopic behavior of integrable interacting systems				
10:00 – 10:30	Break				
Room	Hardy Hall	KMB 211	KMB 212	KMB 213	Clover Hall
Session	TS01-1	TS09-1	TS13-1	TS07-1	TS14
Chair	Teruhisa Tsuda	Walter Van Assche	Margit Rösler	Sho Kubota	Luc Vinet
10:30 – 12:00	**Deniz Bilman** Extreme superposition: rogue waves of infinite order, universality, and anomalous temporal decay	**Hélder Lima** Hahn-classical $(r+1)$-fold symmetric r-orthogonal polynomials	**Meri Zaimi** Bivariate polynomials of Tratnik type	**Etsuo Segawa** Interpolating walk between quantum and random walks on finite graphs	**Luc Lapointe** A q-symmetrization Hamiltonian and a family of orthogonal idempotents
	Amari Jaconelli On Fredholm Pfaffians and Riemann-Hilbert problems	**Jorge Arvesú** On families of discrete multiple orthogonal polynomials on a star-like set	**Misael E. Marriaga** Generalized symmetric classical orthogonal polynomials in two variables	**Hanmeng Zhan** Enabling subspace state transfer with quantum coins	**Plaman Iliev** Gaudin models for some classical multivariate distributions
	Han-Han Sheng On the Fredholm determinant solution to some integrable systems	**Juan Antonio Villegas** Jacobi-Piñeiro multiple orthogonal polynomials on the simplex	**Genki Shibukawa** Commutativity, symmetry and Ruijsenaars-type identities	**Taisuke Hosaka** Behavior of quantum walk with many weak bridges	**Taiyo S. Terada** Proof of the Kresch-Tamvakis conjecture
12:00 – 13:30	Lunch				
Room	Hardy Hall (Chair: Yoshitsugu Takei)				
13:30 – 14:30	**Kohei Iwaki:** Bridging Painlevé equations, topological recursion, and exact WKB analysis				
14:30 – 15:00	Break				
Room	Hardy Hall	KMB 211	KMB 212	KMB 213	Clover Hall
Session	TS01-2	TS09-2	TS13-2	TS07-2	TS16-1
Chair	Deniz Bilman	Walter Van Assche	Nicolas Crampé	Hanmeng Zhan	Lidia Fernández
15:00 – 16:30	**Roozbeh Gharakhloo** Combinatorics of Eulerian graphs on Riemann surfaces	**Teresa Laudadio** Gaussian quadrature rules for multiple orthogonal polynomials	**Satoshi Tsuchimi** A generalization of multivariable μ-function and q-Borel summations	**Akash Kalita** Pretty good fractional revival on abelian Cayley graphs	**Iván Area** On g-orthogonal polynomials: a theoretical tool arising from applications
	Takahiko Nobukawa A rank 3 linear q-difference equation admitting $W(E_8^{(1)})$-symmetry	**Amin Faghih** Recurrence updates and downdates for multiple orthogonal polynomials	**Yosuke Kawamoto** Transforms between multivariate Laguerre and Meixner polynomials and their applications to intertwining relations for stochastic processes	**Koushik Bhakta** Pretty good state transfer in Grover walks on graphs	**Nabiullah Khan** New families of the parametric Apostol polynomials and generalized Voigt functions
	Kanam Park A q-middle convolution and the q-Painlevé equation of type $E_6^{(1)}$	**Rostyslav Kozhan** Zeros of multiple orthogonal polynomials	**Lukas Langen** Uniform bounds on the Dunkl kernel	**Manuel Domínguez de la Iglesia** Exact solutions of open quantum Brownian motions on the real line for two-level systems	**Danilo Jr Dela Cruz** The singular values of Lévy's area matrix
16:30 – 16:45	Break				
Session	TS01-3	TS09-3	TS13-3&TS03	TS07-3	TS16-2
Chair	Alexander Its	Walter Van Assche	Nicolas Crampé & Yoshitsugu Takei	Etsuo Segawa	Nicola Mastronardi & Satoshi Tsujimoto
16:45 – 18:15	**Peter D. Miller** Infinite order solutions of the AKNS hierarchy	**Marcus Vaktnäs** Szegő mapping and Geronimus relations for multiple orthogonal polynomials	**Margit Rösler** Littlewood-Paley theory for orthogonal expansions associated with root systems	**Satoshi Watanabe** Analytical asymptotics of quantum spatial search on 2D lattices with regular range-m edges	**Priyanka Majethiya** Approximation of complex-valued functions by rational interpolation operators: theory and applications
	Percy Deift Applications of a commutation formula II	**Tomas Lasic Latimer** Asymptotics of discrete multiple orthogonal polynomials	**Bart Rosenzweig** Small-time and large-time Borel analysis of Schrödinger equations with point interactions	**Shintaro Yamamura** Implementation and performance evaluation of CVaR-QAOA for ternary portfolio optimization	**Manoj Kumar** Approximation properties of fractional Szàsz-Kantorovich operators involving Hermite polynomials
		Maxim Yattselev Strong asymptotics of weighted orthogonal polynomials	**Gergő Nemes** Exact resurgence in exact WKB analysis		**Nicola Mastronardi** Product integration rules for highly oscillatory integrals
18:15 – 19:00	SIAG/OPSF and OPSFA Business Meeting@Hardy Hall				

Day 3 (August 19th)

Day 3	Hardy Hall (Chair: Howard S. Cohl)				
9:00 | 10:00	**Benjamin Eichinger:** Universality limits and beyond				
10:00 | 10:30	Photo Session & Break				
Room	Hardy Hall	KMB 211	KMB 212	KMB 213	Clover Hall
Session	TS01-4	TS09-4	TS11	TS05-1	TS16-3
Chair	Peter D. Miller	Maxim Yattselev	Kazuki Maeda	Hajime Tanaka	Iván Area
10:30 | 12:00	**Alexander Stokes** Discrete Painlevé equations from orthogonal polynomials: non-conjugate translations, nodal curves and exotic symmetry groups	**Adam Doliwa** Non-commutative multiple bi-orthogonal polynomials: formal approach and integrability	**Dustin Newland** On the expected number of real level crossings of random polynomials spanned by OPUC	**Paul Terwilliger** Strengthening the balanced set condition for the distance-regular graph of the bilinear forms	**Jose Arcia Manoleskos** UL and LU factorizations of birth-death processes associated with classical orthogonal polynomials
	Anton Dzhamay Discrete Painlevé equations with constraints: their geometry and symmetry	**Thomas Wolfs** Multiple orthogonal polynomials of derivative type	**Xiang-Ke Chang** Modified Camassa–Holm peakons, extreme degenerations into Toda-type lattices, and interpretations in terms of orthogonality	**Hirotake Kurihara** Imprimitive association schemes and elimination theory	**Lidia Fernández** Approximation and cubature on planar domains via Zernike-like bases
	Hidehito Nagao Padé method and hypergeometric structures in discrete Painlevé-type systems	**Walter Van Assche** Multiple orthogonal polynomials with modified Bessel weights		**Yushan Pan** Irreducible representations of the S_3-symmetric tridiagonal algebra: the complete graph case	
12:00 | 14:30	Lunch Poster Session@B1F lobby				
14:30 | 18:00	Discussion				
18:30 | 20:30	Banquet@Hotel Okura Kyoto				

List of Poster Presentations

Ichi Fujii (PS01-1) Construction of $\mathbb{Z}_2^2$-integrable hierarchy using $\mathbb{Z}_2^2$-graded algebras	**Ryosuke Naganuma** (PS01-2) Construction and classification of multi-lump solutions of the elliptic two-dimensional Toda lattice equation	**Yuichi Ueno** (PS01-3) On the quantization of higher order Painlevé systems	**Deborah Oliveira** (PS02-1) Provable Weak-to-Strong generalization for neural networks using random matrix theory	**Subaru Hamada** (PS04-1) 3-dimensional spin chain model with nearest neighbor couplings derived from 3-variable Krawtchouk polynomials
Yuki Honda (PS04-2) Orthogonal decomposition and mode expansion of gauge fields on S^2 in the presence of background gauge fields	**Yuta Nasuda** (PS04-3) SWKB quantization condition and solvability via classical orthogonal polynomials	**Qian Chen** (PS07-1) Testing k-block-positivity via semidefinite programs and its complexity	**Kosei Nakamura** (PS07-2) Quantum walks on the non-binary Johnson scheme	

Day 4 (August 20th)

Day 4	Hardy Hall (Chair: David Gómez-Ullate)				
9:00 – 10:00	**María Ángeles García-Ferrero:** The taxonomy of exceptional orthogonal polynomials				
10:00 – 10:30	Break				
Room	Hardy Hall	KMB 211	KMB 212	KMB 213	Clover Hall
Session	TS02-1	TS10	TS12-1	TS05-2	TS15-1
Chair	Karl Liechty	Walter Van Assche	Yves Grandati	Hajime Tanaka	Rakesh K. Parmar
10:30 – 12:00	**Leslie Molag** Local universality of determinantal point processes on $\mathbb{C}^d$	**Max van Horssen** Shift operators for non-symmetric Heckman-Opdam polynomials and non-symmetric Macdonald-Koornwinder polynomials	**F. Alberto Grünbaum** Bispectrality and the ad-conditions	**Hideki Matsumura** Hausdorff construction and combinatorial polynomials	**Andrei Martínez-Finkelshtein** Hypergeometric and q-hypergeometric polynomials with finite free convolutions
	Joakim Cronvall A direct approach to soft and hard edge universality for random normal matrices	**Arno Kuijlaars** Matrix valued orthogonal polynomials in random tiling models	**Ignacio N. Zurrián** Equivalences for matrix-valued bispectral polynomials.	**Ryutaro Misawa** Sharp bounds for equi-chordal tight fusion frames and Grassmann designs	**Dmitrii Karp** q-polynomial perturbations of Heine's and Jackson's transformations
	Sampad Lahiry Birth of a gap: critical phenomena in the normal matrix model	**Inés Pacharoni** Matrix-valued Laguerre-type orthogonal polynomials, Darboux transformations and differential operators	**W. Riley Casper** Binomial prolates, tau functions, and point counting	**Junichiro Matsuda** Algebraic quantum graph theory and application to orthogonal polynomials	**Takao Suzuki** An affine Weyl group action on the basic hypergeometric series arising from the Cluster algebra
12:00 – 13:30	Lunch				
Room	Hardy Hall (Chair: Peter D. Miller)				
13:30 – 14:30	**Manuela Girotti:** Random matrices, random particles, random solitons, and soliton gasses				
14:30 – 15:00	Break				
Room	Hardy Hall	KMB 211	KMB 212	KMB 213	Clover Hall
Session	TS02-2	TS08-1	TS12-2	TS05-3	TS15-2
Chair	Lu Wei	Andrei Martínez-Finkelshtein	Satoru Odake	Hajime Tanaka	Taikei Fujii
15:00 – 16:30	**Chenhao Lu** On the eigenvalue rigidity of the Jacobi unitary ensemble	**Vinay Shukla** A Riemann-Hilbert problem for Jacobi-Pineiro orthogonal polynomials	**Robert Milson** Spectral diagrams and exceptional polynomials	**Younggwang Cho** An explicit formula for Koornwinder moments and Rains' positivity conjecture	**Chihiro Sato** Confluent q-Heun equations based on Newton polygon
	Teije Kuijper Hermitian Jacobi polynomials on the simplex	**Vladimir Lysov** Bipolar Green function and the asymptotics of Cauchy biorthogonal polynomials	**Antonio J. Durán** Using quasi-Darboux transformations to construct exceptional matrix polynomials	**Minho Song** Moments and lattice paths for orthogonal polynomials of type R_{II}	**Ayaka Murakami** q-integral transformation of q-Heun equation and its applications
	John J. Lopez Santander On the asymptotic distribution of the largest eigenvalue of unbalanced complex Jacobi ensembles	**Gökalp Alpan** On a Szegő theorem for extremal polynomials	**Yves Grandati** Rational solutions of dressing chains and higher order Painlevé V equations	**Jihyeug Jang** Discriminants of orthogonal polynomials via symmetric functions	**Priyabrat Gochhayat** On zeros of some ${}_2\phi_1$ polynomials associated with q-Meixner polynomials
16:30 – 16:45	Break				
Session	TS02-3	TS08-2	TS12-3	TS05-4	TS15-3
Chair	Leslie Molag	Vladimir Lysov	Robert Milson	Hajime Tanaka	Dmitrii Karp
16:45 – 18:15	**Michael Voit** Dunkl and Bessel processes with drift: Motivation, an application to noncompact free Brownian motions, and some limit results	**Jacob S. Christiansen** Chebyshev polynomials related to Jacobi weights	**David Gómez-Ullate** Recurrence relations for exceptional Legendre polynomials	**Jae-Ho Lee** Two bases for the Onsager Lie algebra $\mathcal{O}$ and their action on finite-dimensional irreducible $\mathcal{O}$-modules	**Taikei Fujii** Linear relations for Kajihara's q-hypergeometric series
	Wangjun Yuan Operator norm bounds for multi-leg matrix tensors and applications to random matrix theory	**Erwin Miña-Díaz** Chebyshev and Faber polynomials on curves with corners and cusps	**Rachel Bailey** A generalization of DEK-type orthogonal polynomials	**Hau-Wen Huang** Verma modules over the universal Askey–Wilson algebra	**Rakesh K. Parmar** Extended Exton's double hypergeometric functions: Integral forms and functional bounds
	Seungjoon Oh Dissipative spectral form factor of the complex elliptic Ginibre ensemble across various non-Hermiticity regimes	**Benedikt Buchecker** Chebyshev polynomials on a Jordan arc			**Pirivina S** Integral representations and functional bounds for the generalized Horn's function

Day 5 (August 21st)

Day 5	Hardy Hall (Chair: Andrei Martínez-Finkelshtein)				
09:00 | 10:00	**Dan Dai:** Beyond Tracy-Widom: Fredholm determinants, integrable PDEs, and large gap asymptotics				
10:00 | 10:30	Break				
Room	Hardy Hall	KMB 211	KMB 212	KMB 213	Clover Hall
Session	TS02-4	TS08-3	TS04-1	TS06-1	TS15-4
Chair	Johan Kuijper	Hideshi Yamane	Sarah Post	Shuhei Kamioka	Takao Suzuki
10:30 | 12:00	**Victor Alves** Deformations of OP ensembles at a regular bulk point	**Árpád Baricz** Infinitely divisible modified Bessel distributions	**Luc Vinet** The dynamical algebra of the generic superintegrable model on the two-sphere	**Soichi Okada** Universal rational Q-functions	**Shunya Adachi** Asymptotic analysis of the confluent Dotsenko–Fateev equation
	Yeong-Gwang Jung q-deformation of the Marchenko-Pastur law	**Alexander Dyachenko** On vector and branching continued fractions	**Giovanni Rastelli** Symmetry operators and Staeckel separation of Dirac's equation on 3-dimensional spin manifolds	**Masaki Kato** Multiple polylogarithms, symmetric functions and discrete integrable systems	**Saiei-Jaeyeong Matsubara-Heo** Irregular complex hypergeometric integrals
	Lu Wei Average entropy of Bogoliubov-Kubo-Mori random state ensemble		**Kohei Noda** Two-dimensional Coulomb gases with rotation-invariant potentials with singularities	**Stuti Mohanty** Characterization of non-singular hyperplanes of $\mathcal{H}(s,q^2)$ in $\mathrm{PG}(s,q^2)$	**Richard Mikaël Slevinsky** Matrix equations and orthogonal polynomials
12:00 | 13:30	Lunch				
Room	Hardy Hall (Chair: Luc Lapointe)				
13:30 | 14:30	**Rinat Kedem:** q-Whittaker and affine Laumon functions from integrable quiver mutations				
14:30 | 15:00	Break				
Room	Hardy Hall	KMB 211	KMB 212	KMB 213	Clover Hall
Session	TS02-5	TS08-4	TS04-2	TS06-2	TS15-5
Chair	Joakim Cronvall	Árpád Baricz	Hiroshi Miki	Takafumi Mase	Shunya Adachi
15:00 | 16:30	**Karl Liechty** Multiple orthogonal polynomials appearing in the six-vertex model with symmetries	**Hideshi Yamane** Asymptotic expansions of finite Hankel transforms and the surjectivity of convolution operators	**Umme Zainab** Integral equations for Sheffer-Bessel functions through umbral operator	**Nobuhiro Asai** A two-parameter deformation of the Charlier polynomials: bridging combinatorics and probability	**Kerstin Jordaan** Single-point completed interlacing for orthogonal and non-orthogonal polynomial families
	Eui Yoo Free energy expansion of determinantal Coulomb gases in the quadratic fields with a point charge	**Lennart A. Huebner** Asymptotics of weighted Bergman polynomials	**Sarah Post** Exceptional orthogonal polynomials, exactly solvable quantum systems and their deformations		**Vikash Kumar** Two-point completed interlacing and cross-family Wronskian inequalities for related polynomial sequences
	Jiahao Du Asymptotics of the partition function with a one-cut singular potential at the edge	**Howard S. Cohl** Orthogonality relations for the symmetric polynomials in the q^{-1}-Askey scheme			
16:30 | 17:00	Closing				

List of Participants

Shunya Adachi (Utsunomiya University, Japan)
Kanae Akaiwa (Kyoto Sangyo University, Japan)
Gökalp Alpan (Sabancı University, Turkey)
Victor Julio Alves de Souza (Fudan University, China)
Jose Arcia Manoleskos (Universidad de Sevilla, Spain)
Iván Area (Universidade de Vigo, Spain)
Jorge Arvesú (University Carlos III of Madrid, Spain)
Nobuhiro Asai (Aichi University of Education, Japan)
Rachel Bailey (Bentley University, USA)
Árpád Baricz (Babes-Bolyai University and Obuda University, Romania)
Koushik Bhakta (Indian Institute of Technology Guwahati, India)
Deniz Bilman (University of Cincinnati, Turkey)
Dom Vito Briones (Tohoku University, Japan)
Benedikt Buchecker (KU Leuven, Belgium)
Guido Carlet (Université Bourgogne Europe, France)
William Riley Casper (California State University Fullerton, USA)
Ada Chan (York University, Canada)
Xiang-Ke Chang (Academy of Mathematics and Systems Science, Chinese Academy of Sciences, China)
Qian Chen (The Chinese University of Hong Kong, Shenzhen, China)
Shuo Chen (Kyoto University, Japan)
Zean Chen (Kyoto University, Japan)
Younggwang Cho (Sungkyunkwan University, Republic of Korea)
Jacob Stordal Christiansen (Lund University, Sweden)
Howard Cohl (NIST, USA)
Nicolas Crampé (CNRS, France)
Joakim Cronvall (KU Leuven, Belgium)
Dan Dai (City University of Hong Kong, Hong Kong)
Percy Deift (NYU, USA)
Danilo Jr Dela Cruz (University of Oxford, UK)
Deborah Sampaio Antunes de Oliveira (McGill University, Canada)
Adam Doliwa (University of Warmia and Mazury, Poland)
Manuel Domínguez de la Iglesia (Universidad de Alcalá, Spain)
Antonio J Durán (Universidad de Sevilla, Spain)
Alexander Dyachenko (Keldysh Institute of Applied Mathematics, RAS, Russia)
Anton Dzhamay (Beijing Institute of Mathematical Sciences and Applications (BIMSA), China)
Benjamin Eichinger (Lancaster University, UK)
Amin Faghih (KU Leuven, Belgium)
Lidia Fernández (Universidad de Granada, Spain)
Ichi Fujii (Osaka Metropolitan University, Japan)
Taikei Fujii (Ochanomizu University, Japan)
Atsushi Fukuda (Kyoto University, Japan)
María Ángeles García-Ferrero (ICMAT, Spain)
Roozbeh Gharakhloo (University of California Santa Cruz, USA)
Manuela Girotti (Emory University, USA)
Priyabrat Gochhayat (Sambalpur University, Odisha, India, India)
Yves Grandati (Université de Lorraine, France)
F. Alberto Grünbaum (UC Berkeley, USA)
Davide Guzzetti (SISSA, Italy)
David Gómez-Ullate (IE University, Spain)
Subaru Hamada (Doshisha University, Japan)
Yuki Honda (Yokohama National University, Japan)
Taisuke Hosaka (Yokohama National University, Japan)
Hau-Wen Huang (National Central University, Taiwan)
JiSun Huh (University of Seoul, Republic of Korea)
Lennart Hübner (KU Leuven, Belgium)
Plamen Iliev (Georgia Institute of Technology, USA)

Alexander Its (Indiana University Indianapolis, USA)
Kohei Iwaki (The University of Tokyo, Japan)
Amari Ernesta Michela Jaconelli (University of Bristol, UK)
Jihyeug Jang (University of Geneva, Switzerland)
Du Jiahao (City university of hong kong, China)
Kerstin Jordaan (University of South Africa, South Africa)
Yeong-Gwang Jung (Seoul National University, Republic of Korea)
Akash Kalita (Indian Institute of Technology Guwahati, India)
Shuhei Kamioka (Osaka Seikei University, Japan)
Dmitrii Karp (Holon Institute of Technology, Israel)
Masaki Kato (Ritsumeikan University, Japan)
Keigo Kato (Doshisha University, Japan)
Yosuke Kawamoto (Okayama University, Japan)
Rinat Kedem (University of Illinois at Urbana-Champaig, USA)
Jang Soo Kim (Sungkyunkwan University, Republic of Korea)
Koichi Kondo (Doshisha University, Japan)
Rostyslav Kozhan (Uppsala University, Sweden)
Sho Kubota (Aichi University of Education, Japan)
Arno Kuijlaars (Katholieke Universiteit Leuven, Belgium)
Teije Johan Kuijper (Aarhus University, Denmark)
Manoj Kumar (SR University, India)
Vikash Kumar (University of South Africa, South Africa)
Hirotake Kurihara (Yamaguchi University, Japan)
Sampad Lahiry (KU Leuven and University of Melbourne, Belgium)
Lukas Langen (Paderborn University, Germany)
Luc Lapointe (Universidad de Talca, Chile)
Tomas Lasic Latimer (The University of Sydney, Australia)
Teresa Laudadio (Consiglio Nazionale delle Ricerche, Italy)
Jae-Ho Lee (University of North Florida, USA)
Hojoon Lee (Sungkyunkwan University, Republic of Korea)
Karl Liechty (DePaul University, USA)
Hélder Lima (Universidade de Aveiro, Portugal)
Jing Liu (Shandong University, China)
John Jairo Lopez Santander (University of Manitoba, Canada)
Chenhao Lu (City University of Hong Kong, Hong Kong)
Vladimir Lysov (Keldysh Institute of Applied Mathematics, Russia)
Kazuki Maeda (University of Fukuchiyama, Japan)
Priyanka Anilkumar Majethiya (Sardar Vallabhbhai National Institute of Technology (SVNIT), India)
Misael Enrique Marriaga (Universidad Rey Juan Carlos, Spain)
Andrei Martínez-Finkelshtein (Baylor University, USA)
Kenichi Maruno (Waseda University, Japan)
Katsuya Masaki (Doshisha University, Japan)
Takafumi Mase (Kyoto University, Japan)
Nicola Mastronardi (Consiglio Nazionale delle Ricerche, Italy)
Saiei-Jaeyeong Matsubara-Heo (Tohoku University, Japan)
Junichiro Matsuda (Seoul National University, Republic of Korea)
Hideki Matsumura (Tokyo Metropolitan University, Japan)
Takuya Matsuo (University of Fukuchiyama, Japan)
Katsuma Matsushita (Kyoto University, Japan)
Hiroshi Miki (Doshisha University, Japan)
Peter David Miller (University of Michigan, USA)
Robert Milson (Dalhousie University, Canada)
Ryutaro Misawa (Tohoku University, Japan)
Erwin Miña-Díaz (The University of Mississippi, USA)
Stuti Mohanty (National Institute of Technology Rourkela, India)
Leslie Molag (Carlos III University of Madrid, Spain)
Ayaka Murakami (Ochanomizu University, Japan)
Atsushi Nagai (Tsuda University, Japan)

Ryosuke Naganuma (Waseda University, Japan)
Hidehito Nagao (Gifu Shotoku Gakuen University, Japan)
Yusuke Nakahashi (Doshisha University, Japan)
Kosei Nakamura (Doshisha University, Japan)
Yuta Nasuda (National Institute of Technology (KOSEN), Gunma College, Japan)
Gergő Nemes (Harbin Institute of Technology, China)
Dustin William Newland (Indiana University Indianapolis, USA)
Seishiro Nezasa (Doshisha University, Japan)
Takahiko Nobukawa (Kogakkan University, Japan)
Kohei Noda (UCLouvain, Belgium)
Yasuhide Numata (Hokkaido University, Japan)
Satoru Odake (Shinshu University, Japan)
Daniel Ofner (KTH, the royal institute of technology, Sweden)
Hiroshi Ogawara (Josai University, Japan)
Seungjoon Oh (Seoul National University, Republic of Korea)
Ryo Ohkawa (Osaka Metropolitan University, Japan)
Yasuhiro Ohta (Kobe University, Japan)
Soichi Okada (Nagoya University, Japan)
Yasunori Okada (Chiba University, Japan)
Toshio Oshima (University of Tokyo, Japan)
Inés Pacharoni (Universidad Nacional de Cordoba, Argentine)
Yushan Pan (Tohoku University, Japan)
Kanam Park (National Institute of Technology, Toba College, Japan)
Rakesh Kumar Parmar (Pondicherry University (A Central University), India)
Giovanni Rastelli (University of Turin, Italy)
Bart Rosenzweig (The Ohio State University, USA)
Margit Rösler (Paderborn University, Germany)
Pirivina S (Pondicherry University, India)
Hidetaka Sakai (University of Tokyo, Japan)
Makiko Sasada (The University of Tokyo, Japan)
Chihiro Sato (Ochanomizu University, Japan)
Ryusei Sato (Yokohama National University, Japan)
Akari Sato (Doshisha University, Japan)
Etsuo Segawa (Yokohama National University, Japan)
Motoki Seki (Independent, Japan)
Hanhan Sheng (City University of Hong Kong, China)
Genki Shibukawa (Kitami Institute of Technology, Japan)
Naoki Shikakume (Kyoto University, Japan)
Ryosuke Shintaku (Doshisha University, Japan)
Vinay Shukla (Shanghai Jiao Tong University, China, China)
Guilherme Silva (Universidade de São Paulo, Brazil)
Richard Mikaël Slevinsky (University of Manitoba, Canada)
Minho Song (Yonsei University, Republic of Korea)
Alexander Henry Stokes (Waseda University, Japan)
Shinnosuke Sugahara (Kyoto University, Japan)
Takao Suzuki (Kindai University, Japan)
Hiroto Tajika (Kyoto University, Japan)
Ikumi Takayama (Doshisha University, Japan)
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This program is supported by a subsidy funded by the accommodation tax from Kyoto City and the Kyoto Convention & Visitors Bureau.

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Access to the Venue

The conference venue is **Hardy Hall (in Kambaikan), Muromachi Campus, Doshisha University**.

- The venue opens at 8:30 (except for Day 1).
- On Day 1, the reception desk will open around 12:00.

It is just a short walk from **Imadegawa Station (K06) on the Karasuma Subway Line**.

From Kyoto Station to the venue:

1. Take the Karasuma Subway Line **bound for Kokusaikaikan from Platform 2** at Kyoto Station (K11).
   - Besides the Subway, Kyoto Station has separate ticket gates for JR and Kintetsu. You cannot move between different lines inside the gates, so please ensure you enter through the Subway ticket gates. In particular, please note that JR also has underground ticket gates.
2. Ride for 5 stops and get off at Imadegawa Station (K06).
3. From **Exit 2** of Imadegawa Station, walk a short distance north, and you will arrive at the venue. See also the next page.

- Fare: 260 yen
  - You will need to pay with cash to buy a paper ticket, or use a Japanese transportation IC card (such as Suica, ICOCA, or PASMO). Please note that **credit cards cannot be used, either at the ticket gates or to purchase tickets at the machines**.
- Please be very careful not to take a train heading in the opposite direction, as some trains go all the way to Nara[1]!

The process is basically the same if you board from Shijo Station (K09) or Karasuma Oike Station (K08). The fare from Karasuma Oike Station is 230 yen.

Taking the Bus:
Depending on where you are staying, taking the bus might be a convenient option. Please make sure to check the bus route number and destination displayed on the front of the bus.
The nearest bus stop to the venue is Karasuma Imadegawa. The fare is 230 yen, which you pay when exiting the bus. The accepted payment methods are the same as the subway (cash or IC cards), but please note that 5,000-yen and 10,000-yen bills cannot be used on the bus (all coins and 1,000-yen bills are accepted).

- SUBWAY/BUS Navi 1 (Route Map)
- SUBWAY/BUS Navi 2 (How to Ride)

Convenience Stores around the Venue

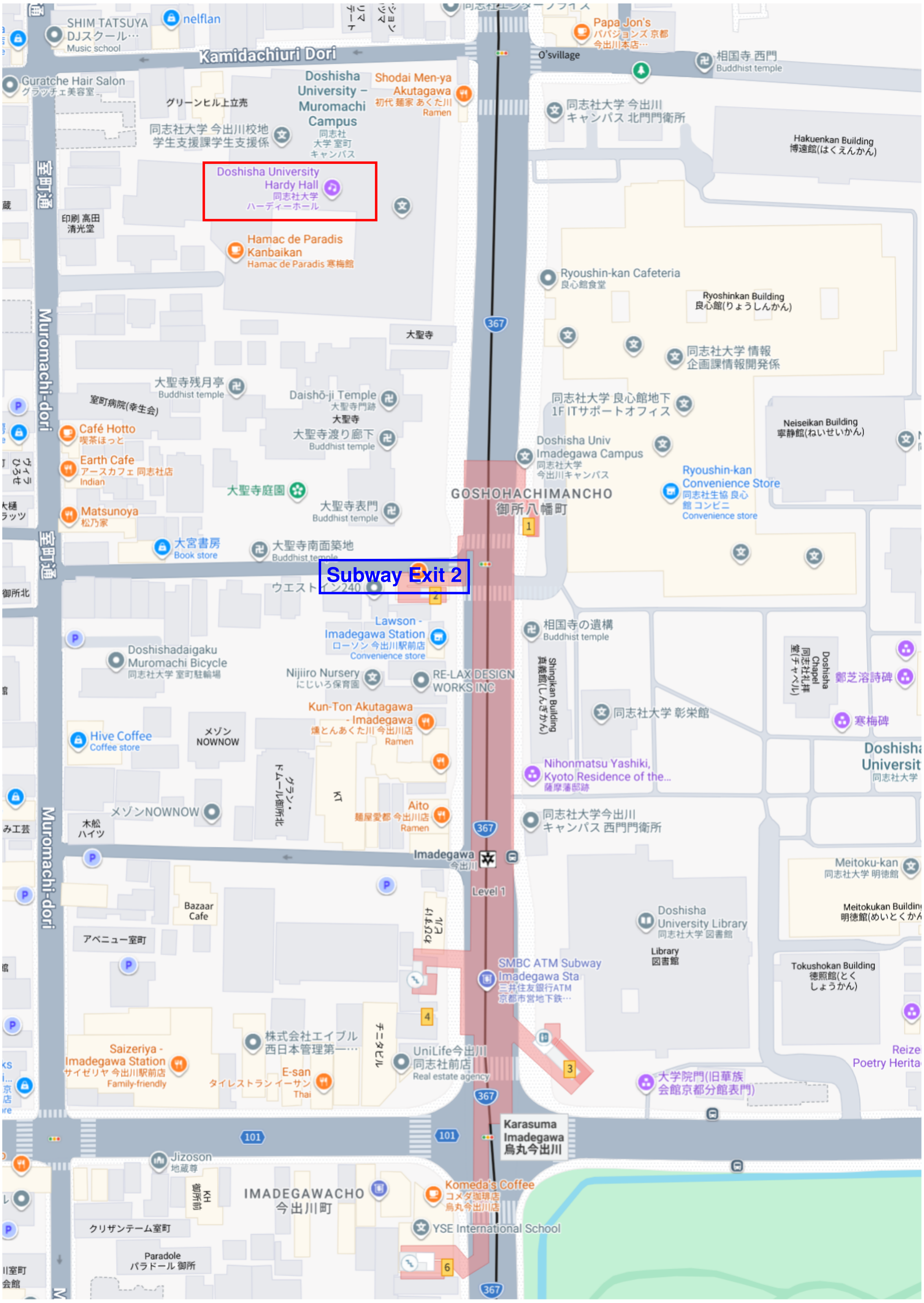
- LAWSON: near the Subway Exit 2
- 7-Eleven: near the Subway Exit 4
- FamilyMart: near the Subway Exit 6

ATMs around the Venue

There are several automated teller machines in the Imadegawa Station, near the Subway Exit 4, and near the Subway Exit 6. You can also find ATMs inside LAWSON, 7-Eleven and FamilyMart.
In Japan, although credit cards can be used in many places, cash is still needed in some situations, such as for the subway and bus systems mentioned above. If you need cash, you can withdraw it from ATMs using a credit card or debit card (service charges or transaction fees may apply).

[1] Nara, an ancient capital famous for its Great Buddha and friendly free-roaming deer, is located 40 km away from Kyoto. While another Doshisha University campus (Kyotanabe Campus) is located along the same train line, it is quite far from the event venue.

Map around the Venue

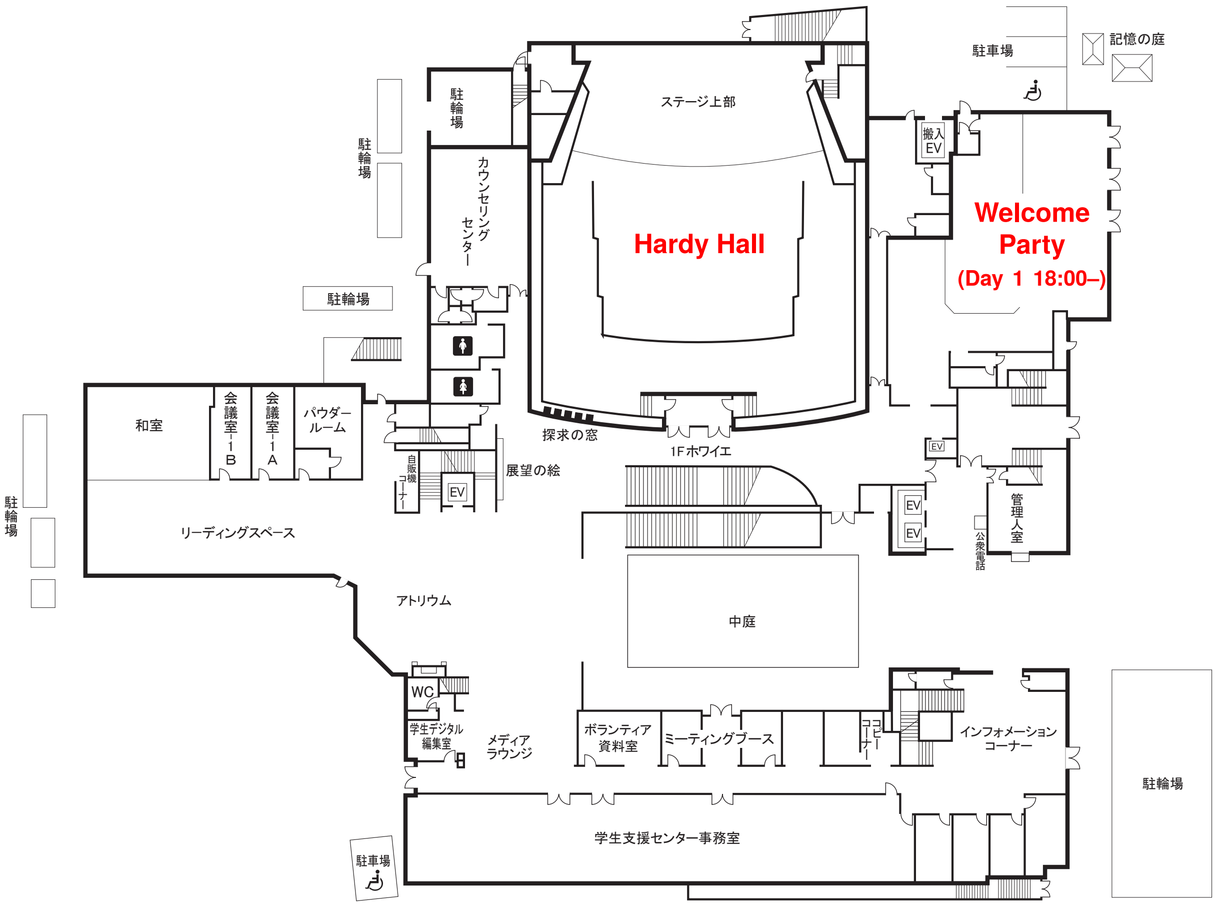


Venue Guidelines (Important!)

- **Only bottled drinks with screw caps may be brought inside the venue. Beverages in disposable cups with lids are not allowed!** This applies **not only to Hardy Hall, but to all rooms of the venue**. We sincerely apologize for any inconvenience and **will instead provide complimentary bottled beverages**.
- **Eating is permitted only in Room A on B1F. Eating in any other room is strictly prohibited.**
- Smoking is strictly prohibited throughout the entire building.

Venue Floor Map

The reception desk is located in the B1F lobby.



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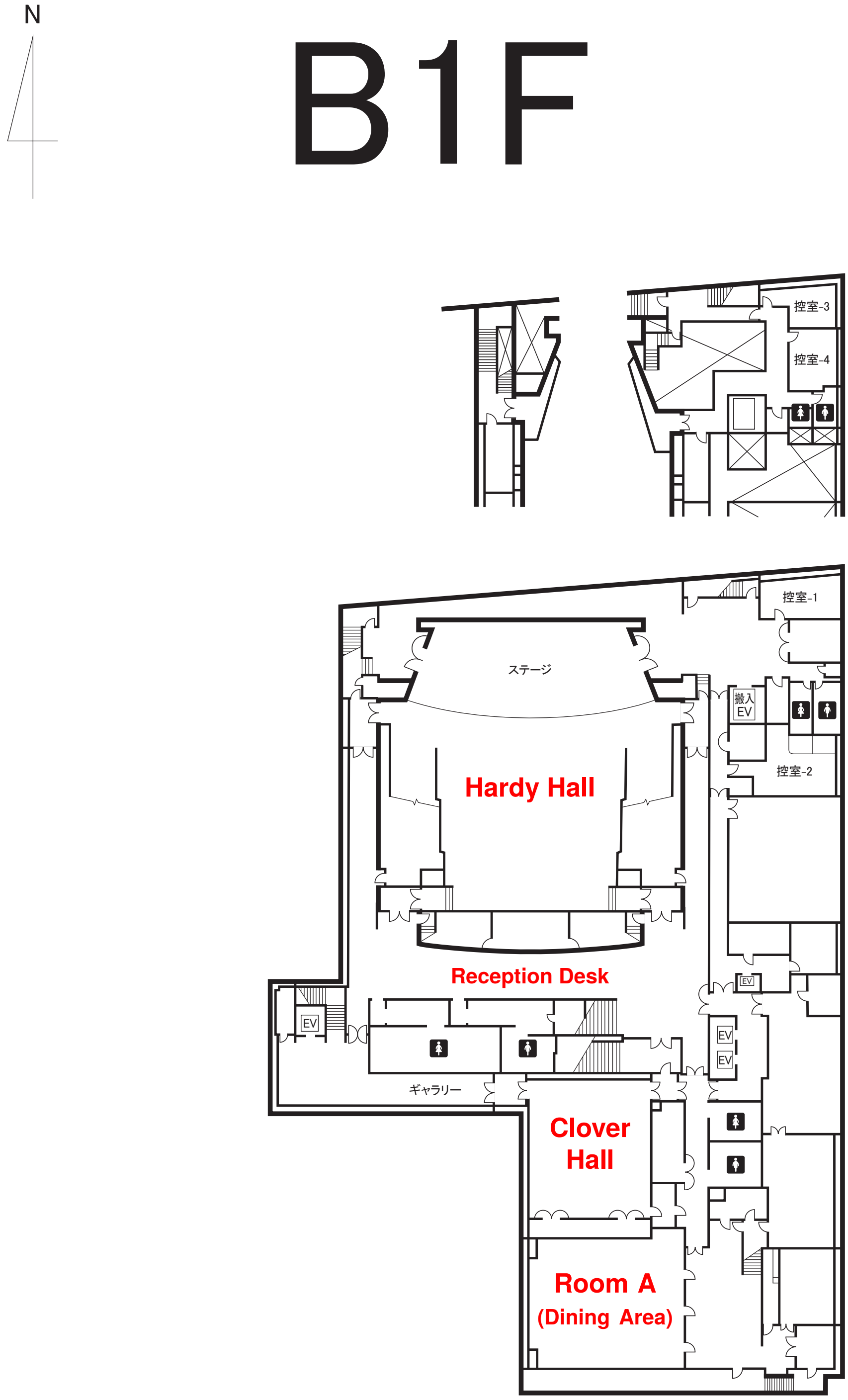
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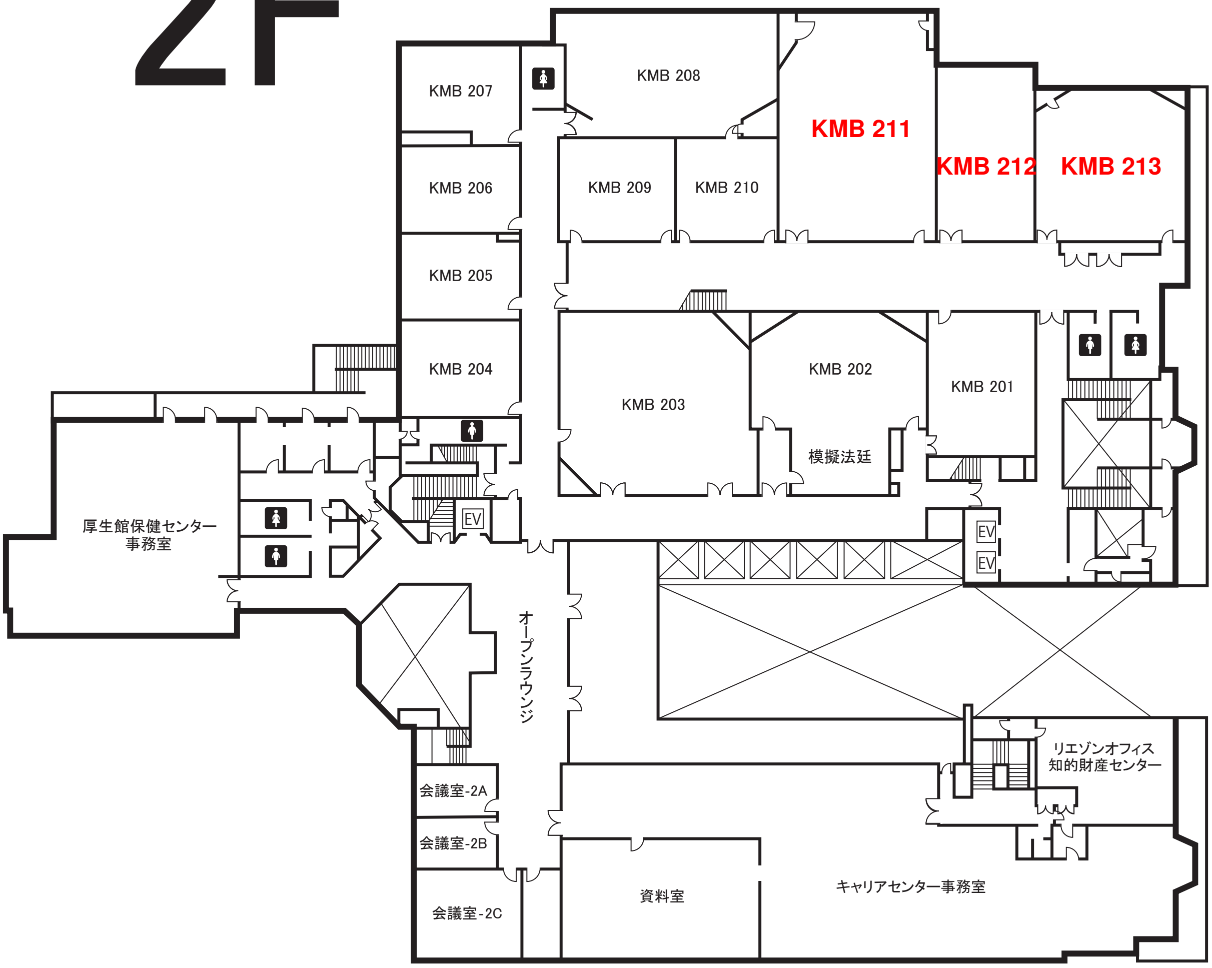
Clover Hall
Room A
(Dining Area)

2F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オープンラウンジ
リエゾンオフィス
知的財産センター
会議室-2A
会議室-2B
会議室-2C
資料室
キャリアセンター事務室

Information

Summer Weather in Kyoto

Summers in Japan have become extremely hot in recent years. In Kyoto, midday temperatures frequently reach 36°C (97°F). Due to high humidity, it often feels much hotter than the actual temperature, increasing the risk of heatstroke. Heatstroke can develop unnoticed until symptoms suddenly become severe. **Please drink water frequently and take care of your health.** We highly recommend avoiding prolonged outdoor walks during the middle of the day.

Welcome Party

The party will take place in the 1st-floor restaurant Hamac de Paradis starting at 18:00 on Day 1. There is no extra charge and participation is optional, so please feel free to join us.

Contributed Talks

Contributed talks will be 30 minutes long. Please present your slides using an HDMI connection. Whiteboards are also available for use in all rooms.

Posters

A0 poster boards will be provided in the B1F lobby. Please display your poster in the assigned location.

Lunch

Lunch boxes will be provided every day, except for Day 1, in Room A for attendees who requested them at registration. The lunch menu changes every day and options will be provided based on your dietary preferences and restrictions. Staff will scan the QR code on your name badge when distributing meals, so please present it at the counter. **Please strictly keep all eating to Room A.**

If you did not request lunch boxes, please visit Hamac de Paradis on the 1st floor (also the Welcome Party venue), or other nearby dining spots and convenience stores. Please be aware that credit cards are not always accepted at ramen shops and small local eateries. Hamac de Paradis might also not take credit cards.

Breaks, Snacks and Drinks

Room A will remain open outside of lunch hours. Snacks and bottled beverages will be available, so please feel free to use this space to take a break. As a reminder, **all snacks must be eaten strictly inside Room A**.

Wi-Fi Access

Details will be provided at the venue.

Free Discussion Rooms

Rooms SK101–SK105 and SK111 in the Shikokan Building, located further northeast from the venue (see also the next page), will be open throughout the conference. Please feel free to use these spaces for open discussion and networking.

Survey / Questionnaire

Survey staff commissioned by our sponsor, the Kyoto Convention & Visitors Bureau, may approach you to complete a short tablet survey. Staff members will be wearing Kyoto Convention & Visitors Bureau armbands. We would greatly appreciate your cooperation.

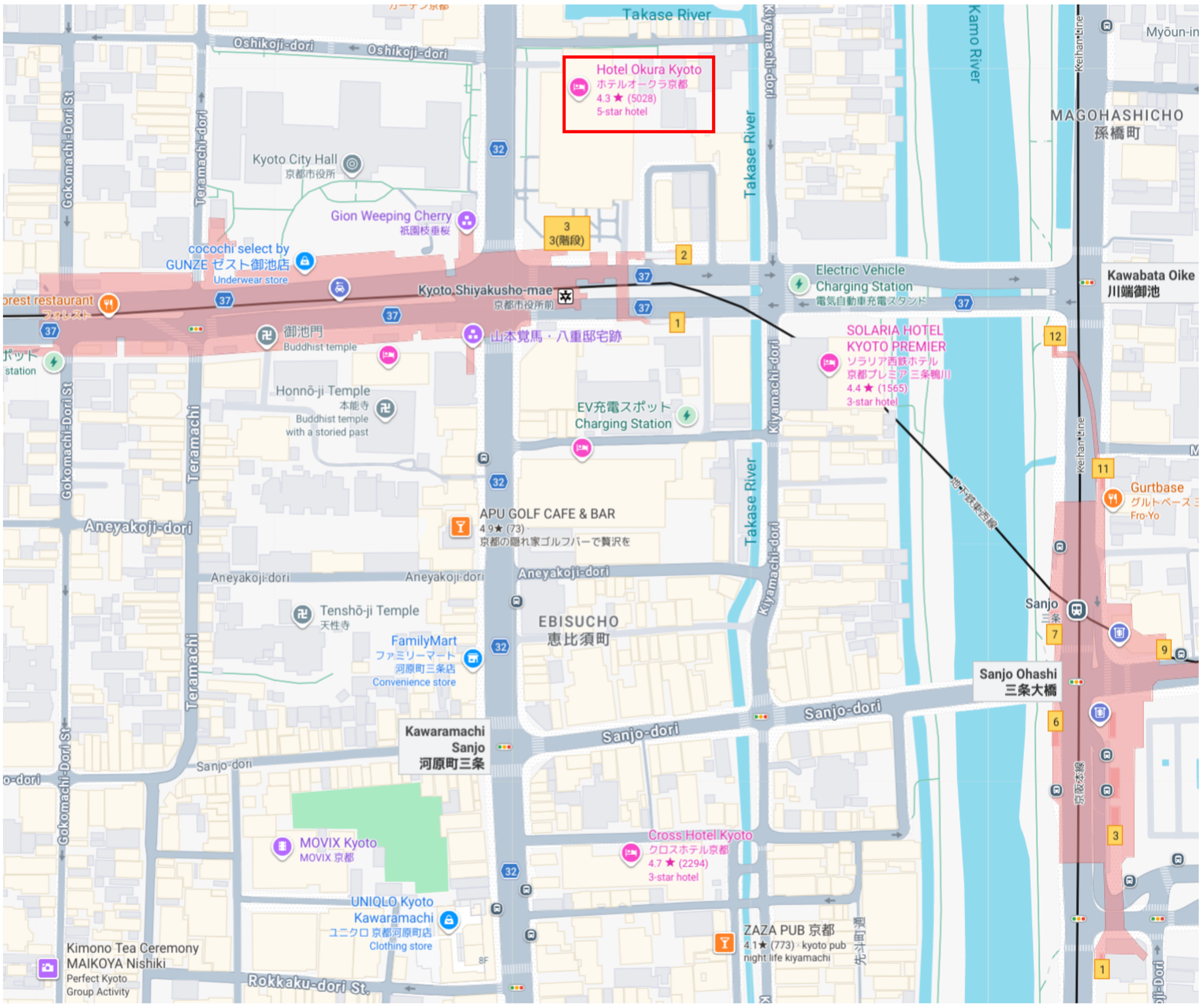
Map to Shikokan Building (Free Discussion Rooms)

Banquet Venue

The banquet will be held at **Hotel Okura Kyoto (SUIUN Banquet Hall, 3rd Floor) on Day 3 starting from 18:30**. Please note that 18:30 is the official start time. If you plan to attend, please arrive a little early.

Transportation to the banquet venue:

- By Subway: Get off at Kyoto Shiyakusho-mae Station (T12) on the Tozai Line and take Exit 3. If taking the Karasuma Line, please transfer at Karasuma Oike Station (K08).
- By Bus: Get off at Kyoto Shiyakusho-mae bus stop.
- By Keihan Railway: Get off at Sanjo Station (KH40) and walk a short distance from there.



Plenary Lectures

Plenary 1: Day 1, 14:30–15:30, Hardy Hall (Chair: Luc Vinet)

Spectrum of quantum relativistic Toda molecule via Hall-Littlewood polynomials

Jan Felipe van Diejen

Universidad de Talca, Talca, Chile

Email: diejen@utalca.cl

The (periodic) Toda chain is a ubiquitous integrable system modeling particles on the line governed by a Hamiltonian of the form

$$H = \sum_{1\le j\le n} \left(\frac{p_j^2}{2} + g_j^2 e^{-(x_j - x_{j+1})} \right),$$

with $g_j \in \mathbb{R}$ and $x_{n+1} \equiv x_1$. For non-vanishing coupling parameters g_j, the energy conservation forces the particles to cluster as a molecule built of n atoms. If one of the couplings is set to zero, the repulsion between neighboring atoms causes the disintegration of the molecule. (When more couplings vanish the Toda chain is seen to decompose into several independently disintegrating Toda molecules.) To date around four decades ago, Ruijsenaars notably found a relativistic deformation of the Toda chain that preserves its integrability. At the level of quantum mechanics, the Hamiltonian of Ruijsenaars' relativistic Toda chain is given by a q-difference analog of the Schrödinger operator for the usual (=non-relativistic) quantum Toda chain. The main goal of this presentation is to point out how Hall-Littlewood polynomials can be employed to compute the spectrum of a lattice version of Ruijsenaars' quantum relativistic Toda molecule.

Plenary 2: Day 1, 15:45–16:45, Hardy Hall (Chair: Teruhisa Tsuda)

Quantum discrete isomonodromic deformations

Yasuhiko Yamada

Kobe University, Kobe, Japan

Email: yamaday@math.kobe-u.ac.jp

The Painlevé equations, or isomonodromic deformations more generally, are important class of nonlinear equations. For the differential Painlevé equations, there is a natural quantization due to their Hamiltonian structure. Some discrete Painlevé equations also have good quantizations. The aim of this talk is to study special solutions for such classical/quantum and continuous/discrete Painlevé equations. Our approach is based on the Padé approximation or interpolation [1]. By setting an appropriate Padé problem, one can obtain the Lax pair and special solutions simultaneously. This method is easy and quick to use, which is a cheeper version of the high class method using the orthogonal polynomials and Riemann-Hilbert problem, see e.g. [2].

As an example we consider the case q-$P_{\rm VI}$. Let $P_m(x), Q_n(x)$ be the polynomials of degree m, n such that $\psi(x) := \prod_{i=1}^{2} \frac{(a_i x;q)_\infty}{(b_i x;q)_\infty} = \frac{P_m(x)}{Q_n(x)} + O(x^{m+n+1})$. Then for $y = P_m(x), \psi(x)Q_n(x)$ we have

$$
\begin{aligned}
L_1 :=&\Big\{\frac{x(f-a_2)(f-b_2)q^{m+n+1}}{fg(fx-q)} + \frac{gx(f-a_1)(f-b_1)}{f(fx-1)} + \frac{x(a_1a_2q^m + b_1b_2q^n)}{f} - q^{m+n+1} - 1\Big\} y(x) \\
&- \frac{(a_1x-1)(a_2x-1)}{fx-1} y(qx) + \frac{(q-b_1x)(q-b_2x)q^{m+n}}{q-fx} y(\frac{x}{q}) = 0, \\
L_2 :=&(1-a_2x)y(qx) + g(b_1x-1)y(x) - (1-g)(1-fx)Ty(x) = 0.
\end{aligned}
$$

This is a Lax pair for q-$P_{\rm VI}$ for the unknown variables f, g, where $T = T_{a_1,q}T_{b_1,q}$ is the discrete time evolution $T_{a,q} : a \to qa$. Taking a suitable linear combination of L_1, L_2, we get

$$
\begin{aligned}
QP_{\rm VI} :=&(g-1)(q-1)\Big\{(a_1x-1)Ty(x) + y(x)\Big\} - \Big\{x^2(a_1a_2q^m + b_1b_2q^n) + ux + (1+q^{m+n+2})\Big\} y(x) \\
&+ (a_1x-1)(a_2x-1)y(qx) + (q-b_1x)(q-b_2x)q^{m+n} y(\frac{x}{q}) = 0, \qquad (1)
\end{aligned}
$$

where the variables f, g are almost put away into a single parameter u. This non-stationaly q-Heun equation can be viewed as a quantum q-$P_{\rm VI}$ equation. In fact, through a certain $q \to 1$ limit, this equation gives known quantization of $P_{\rm VI}$ (or BPZ equation for Virasoro CFT). By the construction, we have explicit special solutions. In case of $(m, n) = (3, 2)$ for example, the solution is given by

$$
P(x) = s_{(3,3)} + s_{(3,3,1)}x + s_{(3,3,2)}x^2 + s_{(3,3,3)}x^3, \qquad Q(x) = s_{(3,3)} - s_{(4,3)}x + s_{(4,4)}x^2,
$$

where $s_\lambda = \det\left(p_{\lambda_i - i + j}\right)_{i,j=1}^{\ell(\lambda)}$ and $\psi(x) = \sum_{k=0}^{\infty} p_k x^k$, i.e. $p_k = \frac{b_2^k(\frac{a_2}{b_2})_k}{(q)_k} {}_2\varphi_1\left(q^{-k}, \frac{a_1}{b_1}, q^{-k+1}\frac{b_2}{a_2}; q\frac{b_2}{a_1}\right)$.

After the above review, we will discuss some related developments: (i) The zeros of the polynomials P and Q, viewing as a polynomial in x (with parameter t) or as a polynomial in t (with parameter x). In particular, we study the case of differential $P_{\rm IV}$, where the τ-functions of some special solutions are given by the generalized Hermite polynomials $H_{m,n}(t)$ and the structure of their zeros has been investigated in detail. Some interesting dynamics of the zeros $x_i(t)$ or $t_j(x)$ will be observed. (ii) The quantization of the q-FST system. The q-FST system was obtained by Tsuda and Fuji-Suzuki independently as rank N generalization of the q-$P_{\rm VI}$ (corresponding to $N = 2$). In a series of works [3], we proposed certain non-stationary equation as the quantization of q-FST system and conjectured (and proved for $N = 2$) that the partition function of $\widehat{\mathfrak{gl}}_N$ affine Laumon space solves the equation. Since the equation contains infinite products, the structure is quite different form usual algebraic q-difference equations. We will see, however, that the same partition function also satisfy the equation similar to (1) in certain special situation.

References

[1] H. Nagao and Y. Yamada, *Padé Methods for Painlevé equations*, Springer Briefs **42** (2021).

[2] W. Van Assche, *Orthogonal Polynomials and Painlevé Equations*, Australian Math. Soc. Lecture Series **27** (2017).

[3] H. Awata, K. Hasegawa, H. Kanno, R. Ohkawa, S. Shakirov, J. Shiraishi and Y. Yamada, *Non-stationary difference equation and affine Laumon space*: I, SIGMA **19** (2023), 089; II, SIGMA **20** (2024), 077; III, preprint [arXiv:2510.27142].

Plenary 3: Day 1, 17:00–18:00, Hardy Hall (Chair: Inés Pacharoni)

Orthogonal polynomials at finite temperature

Guilherme Silva

Universidade de São Paulo, São Carlos, Brazil

Email: silvag@usp.br

Motivated by connections with systems of free fermions at finite temperature, a new class of deformations of orthogonal polynomials has emerged in recent years. Introducing the deformation reshapes the classical picture, and a rich behavior of some structures we rely on, from recurrence relations to asymptotics, emerge. We will give an overview of these developments and show how these deformed orthogonal polynomials offer insight into models in mathematical physics.

Plenary 4: Day 2, 9:00–10:00, Hardy Hall (Chair: Satoshi Tsujimoto)

Stationary distributions and macroscopic behavior of integrable interacting systems

Makiko Sasada

The University of Tokyo, Tokyo, Japan

Email: sasada@ms.u-tokyo.ac.jp

In this talk, we investigate the stationary distributions and macroscopic behavior of interacting systems, presenting insights from two distinct frameworks. First, we study $1 + 1$-dimensional discrete integrable lattice models, such as the box-ball system (BBS), driven by deterministic time evolution. Based on our recent work, we review the properties of their i.i.d. invariant measures and the resulting macroscopic dynamics. Second, we introduce a certain class of interacting particle systems with stochastic time evolution, defined by a polynomial-preserving structure where the Markov generator maps any polynomial of degree n to a polynomial of degree at most n. For this class, we completely characterize all possible i.i.d. invariant measures and derive their large-scale macroscopic behavior. We also provide a rich variety of concrete examples that fit into this framework.

Plenary 5: Day 2, 13:30–14:30, Hardy Hall (Chair: Yoshitsugu Takei)

Bridging Painlevé equations, topological recursion, and exact WKB analysis

Kohei Iwaki

The University of Tokyo, Japan

Email: k-iwaki@g.ecc.u-tokyo.ac.jp

Painlevé equations provide a meeting point for several important subjects in mathematical physics, including orthogonal polynomials, random matrices, integrable systems, topological recursion, and exact WKB analysis. Moreover, the Kyiv formula discovered by Gamayun, Iorgov, and Lisovyy has revealed deep connections with conformal field theory, gauge theory, and topological string theory. As a result, solutions of Painlevé equations are becoming increasingly important as special functions in modern mathematical physics.

In the first part of the talk, I will focus on the relationship between Painlevé equations and topological recursion, and review a construction of tau functions of Painlevé equations based on a discrete Fourier transform, or Zak transform. Our main example will be the first Painlevé equation

$$\hbar^2 \frac{d^2 q}{dt^2} = 6q^2 + t$$

with a small formal parameter $\hbar$. This construction provides a framework for describing τ-functions of Painlevé equations as non-perturbative partition functions in topological recursion:

$$\tau(t,\nu,\rho;\hbar) = \sum_{k\in\mathbb{Z}} e^{2\pi i k\rho/\hbar} Z(t,\nu + k\hbar;\hbar),$$

where

$$Z(t,\nu;\hbar) = \exp\left(\sum_{g\geq 0} \hbar^{2g-2} F_g(t,\nu)\right)$$

is the perturbative partition function obtained by applying the topological recursion to the following family of Torelli-marked elliptic spectral curves:

$$y^2 = 4x^3 + 2tx + u(t,\nu), \qquad \nu = \oint_A y\,dx.$$

I will briefly explain the framework of topological recursion, and then discuss why it is essential in solving Painlevé equations.

In the second part of the talk, I will discuss some of the speaker's results, including conjectural aspects, on the Stokes structure and resurgence properties of these tau functions and partition functions with respect to $\hbar$. A key role in the derivation is played by exact WKB analysis of linear differential equations related to Painlevé equations through isomonodromic deformations. In the speaker's CMP paper published in 2020, the linear Stokes multipliers, which are conserved quantities of Painlevé equations, were described explicitly in terms of the Voros symbols

$$(e^{2\pi i\nu/\hbar}, e^{2\pi i\rho/\hbar})$$

under certain technical assumptions, including the Borel summability of the partition functions. I will explain how, by combining these results with monodromy invariance —that is, the integrability of Painlevé equations— one obtains an explicit description of the Stokes phenomena of the partition functions in terms of cluster transformation. I will also explain how this description leads to formulas predicted by theoretical physicists through the non-perturbative analysis of the holomorphic anomaly equation. The topics presented in the second half of the talk are based on joint work with Marcos Mariño (University of Geneva).

References

[1] Kohei Iwaki, 2-parameter τ-function for the first Painlevé equation: topological recursion and direct monodromy problem via exact WKB analysis, *Comm. Math. Phys.* **377** (2020), 1047–1098, arXiv:1902.06439.

[2] Kohei Iwaki and Marcos Mariño, Resurgent Structure of the Topological String and the First Painlevé Equation, *SIGMA* **20** (2024), 028, 21 pages, arXiv:2307.02080.

Plenary 6 (Gábor Szegő Prize Laureate): Day 3, 9:00–10:00, Hardy Hall (Chair: Howard S. Cohl)

Universality limits and beyond

Benjamin Eichinger

Lancaster University, UK

Email: b.eichinger@lancaster.ac.uk

Universality describes the phenomenon that the local behavior of zeros of orthogonal polynomials depends only on general characteristics of the system, rather than on its precise properties. From a modern perspective, such local behavior is analyzed through local scaling limits of the Christoffel–Darboux kernel. Recent works have established necessary and sufficient conditions for the most studied universality classes, including bulk universality and hard-edge universality. In all these settings, the rescaled Christoffel–Darboux kernel converges to a single limit kernel.

After surveying universality limits with a single limit kernel, we will also consider scaling limits with multiple accumulation points, leading to new universality classes. We show that this typically arises when studying scaling limits for orthogonal polynomials associated with the equilibrium measure of the real Julia set of an expanding polynomial or with the Cantor measure on the classical middle-third Cantor set.

The talk is based on joint work with Alexander Kheifets, Milivoje Lukić, Brian Simanek, Harald Woracek and Peter Yuditskii.

Plenary 7: Day 4, 9:00–10:00, Hardy Hall (Chair: David Gómez-Ullate)

The taxonomy of exceptional orthogonal polynomials

María Ángeles García-Ferrero

ICMAT, Madrid, Spain

Email: garciaferrero@icmat.es

Exceptional orthogonal polynomials arise as eigenfunctions of Sturm-Liouville problems and form complete systems in weighted L^2 spaces. Nevertheless, contrary to the classical polynomials of Hermite, Laguerre and Jacobi, their sets of degrees miss finitely many natural numbers.

In this talk, we will focus on the method for constructing exceptional orthogonal polynomial systems based on Darboux transformations of second-order differential operators. This method actually allows us to construct *all* families of exceptional orthogonal polynomials and thus classify them completely.

Based on past and ongoing work with David Gómez-Ullate and Robert Milson.

Plenary 8: Day 4, 13:30–14:30, Hardy Hall (Chair: Peter D. Miller)

Random matrices, random particles, random solitons, and soliton gasses

Manuela Girotti

Emory University, Atlanta, United States

Email: manuela.girotti@emory.edu

Soliton solutions are special solutions to integrable nonlinear wave equations. Their name, coined by N. Zabusky and M. Kruskal in 1965, evokes the characteristic feature of elastic interaction between such solitary waves, much like particles.

The first part of the talk is a broad introduction to the theory of solitons and integrable PDEs. The second, and main, part of the talk will focus on some new developments and growing interest in a special class of solutions known as ”soliton gas”.

I will describe a collection of works carried out collaboration with K. McLaughlin (Tulane), T. Grava (SISSA/Bristol), R. Jenkins (UCF), A. Minakov (U. Karlova), J. Najnudel (Bristol).

We analyze a large collection of random solitons described in terms of the eigenvalues of random matrices, and the concentration phenomena that ensue as the matrix dimension grows. The limiting *deterministic* solution is an instance of a soliton gas. We then study the gas's dynamics and its long-time behaviour in the presence of a single trial soliton travelling through it: the velocity of such a trial soliton satisfies Generalized Hydrodynamics equations, originally proposed by V. Zakharov and G. El, which are typical of certain many-particle systems.

Plenary 9: Day 5, 9:00–10:00, Hardy Hall (Chair: Andrei Martínez-Finkelshtein)

Beyond Tracy-Widom: Fredholm determinants, integrable PDEs, and large gap asymptotics

Dan Dai

City University of Hong Kong, Hong Kong, China

Email: dandai@cityu.edu.hk

The Tracy-Widom distribution is a classic example of the deep interplay between random matrix theory, in- tegrable systems, and special functions. Defined as the Fredholm determinant of the Airy kernel, it is also characterized by the Painlevé II equation.

In this talk, I will present recent progress extending this framework to families of non-classical kernels beyond the Airy, Bessel, and sine universality classes. These kernels arise naturally in models exhibiting critical phenomena. The unifying methodological tool is the Riemann-Hilbert approach to integrable operators, following the foundational work of Jimbo, Miwa, and Sato. Using this structure, I will show how the corresponding Fredholm determinants are related to nonlinear integrable PDEs–including the KdV, mKdV, and Boussinesq equations–thereby extending the classical connections to Painlevé ODEs. The large gap asymptotics of these determinants will also be discussed.

Plenary 10: Day 5, 13:30–14:30, Hardy Hall (Chair: Luc Lapointe)

q-Whittaker and Affine Laumon functions from integrable quiver mutations

Rinat Kedem

University of Illinois

Email: rinat@illinois.edu

Eigenfunctions of the q-difference (open) Toda Hamiltonians associated with finite-dimensional Lie algebras are called q-Whittaker functions. For any root system of a finite-dimensional Lie algebra $\mathfrak{g}$, there is a quantum cluster algebra and a mutation sequence – a quiver automorphism– which we call a discrete time translation [1, 4], or Dehn twist in type A. In terms of a choice of quantum torus representation of the cluster variables, the time translation operator becomes a q-difference operator $\mathcal{T}_{\mathfrak{g}}$. The q-difference Toda Hamiltonians are elements in the upper cluster algebra which commute with $\mathcal{T}_q$ [2]. The common universal eigenfunctions of the Hamiltonian and time-translation operator can be computed as infinite series in a specific regime, and are unique up to a scalar multiple.

Generalizing the construction of the time translation operator for the cluster algebra corresponding to an affine root system yields a well-defined operator $\mathcal{T}_{\mathfrak{g}}$. Since the Cartan matrix is singular, the eigenfunctions have an essential singularity. For this reason, it is necessary to construct the time-translation operator $\widehat{\mathcal{T}_{\mathfrak{g}}}$ using the extended Cartan matrix, which is non-singular. The closed q-difference Toda Hamiltonian commutes with $\mathcal{T}_{\mathfrak{g}}$ but not with with $\widehat{\mathcal{T}_{\mathfrak{g}}}$. Instead, $\widehat{\mathcal{T}_{\mathfrak{g}}}$ acts as a dynamical symmetry of the Hamiltonian: The Hamiltonian depends on the null root of $\mathfrak{g}$ via a parameter p, and $\widehat{\mathcal{T}_{\mathfrak{g}}}H(p)\widehat{\mathcal{T}_{\mathfrak{g}}}^{-1} = H(\kappa p)$, where κ is a dual parameter. The eigenstates of $\widehat{\mathcal{T}_{\mathfrak{g}}}$ are non-stationary states, called affine Laumon functions. In the $\kappa \to 1$ limit, their essential singularity can be normalized away to obtain eigenfunctions of the closed Toda Hamiltonians.

In the case where $\mathfrak{g} = \widehat{\mathfrak{gl}}_n$, the more general t-deformed operators $\widehat{\mathcal{T}_{\mathfrak{g}}}^t$ were introduced for the Ruijsenaars system in [5], together with conjectures about properties of their eigenfunctions. The procedure above gives the Toda limit $t \to 0$ of these operators.

We can generalize the operators $\widehat{\mathcal{T}_{\mathfrak{g}}}$ and their eigenfunctions to other root systems in the Toda limit using cluster mutation sequences. We give a uniform way to define a cluster algebra (also called the Q-system cluster algebras [1, 4] or Coulomb branch), the discrete difference operators $\widehat{\mathcal{T}_{\mathfrak{g}}}$, and their eigenfunctions for any root system data of a Lie algebra, finite or affine. The generalized mutation sequence corresponding to time translation was given for all finite root systems in [1, 4] and are easily generalized to the affine root systems. These sequences are quiver automorphisms and, in the case of finite root systems, express the integrable structure of a discrete system, with commuting integrals of motion in the upper cluster algebra which are conserved under the mutation sequence [2]. These are the q-deformed (open) Toda Hamiltonians.

The operators $\widehat{\mathcal{T}_g}$ can be used iteratively to construct explicit, Fermionic-type formulas for the q-Whittaker functions. This is a semi-infinite type construction. The resulting formulas have a form which specializes to the characters of principal subspaces introduced in [6] or to the generating functions for Betti numbers of Nakajima quiver varieties [3].

Based on Joint work with M. Bershtein, J-E Bourgine, P. Di Francesco, V. Pasquier, J. Shiraishi.

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Contributed Talks

TS01: Painlevé and Integrable Systems

Coordinators: Alexander Its, Teruhisa Tsuda

TS01-1: Day 2, 10:30–12:00, Hardy Hall (Chair: Teruhisa Tsuda)

TS01-1-1

Extreme superposition: rogue waves of infinite order, universality, and anomalous temporal decay

Deniz Bilman

University of Cincinnati, Cincinnati, United States

Email: bilman@uc.edu

Focusing nonlinear Schrödinger equation serves as a universal model for the amplitude of a wave packet in a general one-dimensional weakly-nonlinear and strongly-dispersive setting that includes water waves and nonlinear optics as special cases. Rogue waves of infinite order are a novel family of solutions of the focusing nonlinear Schrödinger equation that emerge universally in a particular asymptotic regime involving a large-amplitude and near-field limit of a broad class of solutions of the same equation. In this talk, we will present several recent results on the emergence of these special solutions along with their interesting asymptotic and exact properties. Notably, these solutions exhibit anomalously slow temporal decay and are connected to the third Painlevé equation. Finally, we will extend the emergence of rogue waves of infinite order to the first several flows of the AKNS hierarchy—allowing for arbitrarily many simultaneous flows—and report on recent work regarding their space-time asymptotic behavior under a general flow from the hierarchy. This involves joint works with Peter D. Miller, Thierry Laurens, Giorgio Young, and Elliot Blackstone.

TS01-1-2

On Fredholm Pfaffians and Riemann-Hilbert problems

Thomas Bothner, Amari Jaconelli

University of Bristol, Bristol, United Kingdom

Email: thomas.bothner@bristol.ac.uk, amari.jaconelli@bristol.ac.uk

In my recent work with Thomas Bothner, we explore how classes of Fredholm Pfaffians, which can describe the eigenvalue distributions for orthogonal and symplectic matrix ensembles, can be computed in terms of canonical, auxiliary Riemann-Hilbert problems.

We do this by first algebraically manipulating the kernel of the Fredholm Pfaffian and then relating the resulting Pfaffian to a Riemann-Hilbert problem, assuming its main kernel is either of additive Hankel composition or truncated Wiener-Hopf type. One can then find asymptotic results for the Fredholm Pfaffian as a natural consequence of the Rieman-Hilbert problem characterisation.

TS01-1-3

On the Fredholm determinant solution to some integrable systems

Dan Dai, Han-Han Sheng

City University of Hong Kong, Hong Kong, P.R. China

Email: hsheng@cityu.edu.hk

The τ-function, introduced by the Kyoto school, is a central object in the theory of soliton equation hierarchies. In this talk, we present τ-functions for the modified Kadomtsev–Petviashvili (mKP) hierarchy and the two-dimensional Toda lattice (2DTL) hierarchy. These τ-functions are represented as Fredholm determinants of a family of linear operators whose kernels satisfy linear constant-coefficient PDEs in the hierarchy variables. The proof is based on general properties of traces and Fredholm determinants. We will also discuss Fredholm determinant representations of solutions to several classical continuous integrable equations in (1+1)-dimensions, such as the modified KdV equation and the nonlinear Schrödinger equation from the perspective of reductions. Specific examples include solitons, soliton gases, breather gases, and rogue-wave-type solutions. This is joint work with Prof. Dan Dai.

TS01-2: Day 2, 15:00–16:30, Hardy Hall (Chair: Daniz Bilman)

TS01-2-1

Combinatorics of Eulerian graphs on Riemann surfaces

Ahmad Barhoumi[1], Roozbeh Gharakhloo[2], Nathan Hayford[3], Tomas Lasic Latimer[4]

[1]Grinnell College, Grinnell, USA
[2]University of California Santa Cruz, Santa Cruz, USA
[3]KTH Royal Institute of Technology, Stockholm, Sweden
[4]University of Sydney, Sydney, Australia

Email: roozbeh@ucsc.edu

Let $N_g(m, j)$ denote the number of connected labeled m-valent graphs with j vertices whose minimal embedding genus is g. For example, $N_0(4, 1) = 2$ and $N_1(4, 1) = 1$, as can be verified by drawing the connected labeled 4-valent graphs with a single vertex on the sphere and on the torus. The problem of determining $N_g(m, j)$ arose in models of two-dimensional quantum gravity and has since inspired a rich body of work, involving both combinatorial methods and techniques from random matrix theory.

Using connections with orthogonal polynomials, in this talk I will present new results from two recent joint works on explicit formulae for Eulerian, i.e. even-valent, graph counts in which both the valence parameter(s) and the number of vertices enter as variables. The first, with Tomas Lasic Latimer, concerns the enumeration of *regular* Eulerian graphs embedded in compact Riemann surfaces of genus g, for $2 \leq g \leq 4$ and, in principle, beyond. These results extend the fully explicit formulae of Ercolani-McLaughlin-Pierce (2008) for $g = 0$ and of Ercolani-Lega-Tippins (2023) for $g = 1$. We also obtain leading-order large-valence asymptotics of these counts for $g \leq 4$ and formulate a structural conjecture for $g \geq 5$.

The second joint work, with Ahmad Barhoumi and Nathan Hayford, concerns the more intricate combinatorics of Eulerian graphs of mixed valence: more precisely, graphs with n_k vertices of valence $2k$, for $k = 1, \ldots, N$, with arbitrary $N \in \mathbb{N}$. We obtain explicit closed-form formulae for these counts in the planar and toroidal cases.

TS01-2-2

A rank 3 linear q-difference equation admitting $W(E_8^{(1)})$-symmetry

Takahiko Nobukawa

Kogakkan University, Mie, Japan

Email: t-nobukawa@kogakkan-u.ac.jp

In Sakai's theory, discrete Painlevé equations are classified by the type of difference (additive/multiplicative/elliptic) and the affine Weyl group symmetry. Many Painlevé-type equations can be expressed as the compatibility condition of two linear equations (a so-called Lax pair). It is therefore important to construct linear equations admitting affine Weyl group symmetries. Yamada's scalar Lax pair [Yamada (2010, IMRN)] is well-known as a Lax formalism for the q-Painlevé equation of type $E_8^{(1)}$.

In this talk, we introduce a rank 3 q-difference system which admits the affine Weyl group symmetry of type $E_8^{(1)}$. A key feature is the absence of apparent singularities. Since our system is given in matrix form, no apparent singularities arise. We also give a characterization of this system in terms of its spectral type. One of the generators of this Weyl group is described using the q-middle convolution [Sakai–Yamaguchi (2017, IMRN), Arai–Takemura (2025, arXiv:2503.11214)], therefore we explain its formulation and several properties. In addition, we discuss a relationship between the system and Moriyama–Yamada's quantum curve of type $E_8^{(1)}$ [Moriyama–Yamada, (2021, SIGMA)].

We obtain the above system from a "democratic" point of view. Here, the word "democratic" means that every point can be treated equally (see also Yoshida's book [Yoshida, *Hypergeometric Functions, My Love*] for this terminology). The resulting system can be regarded as a "democratic" q-analog of a Fuchsian differential equation of rank three with three singularities. If time permits, we will also explain this idea.

This talk is based on our preprint, arXiv:2601.06070 [math.QA].

TS01-2-3

A q-middle convolution and the q-Painlevé equation of type $E_6^{(1)}$

Kanam Park

National Institute of Technology, Toba College, Mie, Japan

Email: paku-k@toba-cmt.ac.jp

The middle convolution was introduced by Katz and formulated by Dettweiler and Reiter as an operation on Fuchsian differential systems. A q-difference analogue of the middle convolution was introduced by Sakai and Yamaguchi and was later reformulated by Arai and Takemura to provide a natural description of its additivity. Sasaki, Takagi, and Takemura showed that a transformation obtained by applying the q-middle convolution to the linear equation associated with the q-Painlevé VI equation is expressed in terms of the affine Weyl group symmetries of the q-Painlevé VI equation. In this talk, we report on results obtained by applying the q-middle convolution reformulated by Arai and Takemura to the 3×3 matrix linear q-difference equation associated with the q-Painlevé equation of type $E_6^{(1)}$.

TS01-3: Day 2, 16:45–17:45, Hardy Hall (Chair: Alexander Its)

TS01-3-1

Infinite-order solutions of the AKNS hierarchy

Peter D. Miller[1], Deniz Bilman[2], Elliot Blackstone[1], Robert Buckingham[2], Robert Jenkins[3], Liming Ling[4], Giorgio Young[1,5]

[1]University of Michigan, Ann Arbor, USA
[2]University of Cincinnati, Cincinnati, USA
[3]University of Central Florida, Orlando, USA
[4]South China University of Science and Technology, Guangzhou, China
[5]University of Wisconsin, Madison, USA

Email: millerpd@umich.edu

We describe several applications and properties of a class of particular solutions of the partial differential equations of the AKNS hierarchy, for instance the focusing nonlinear Schrödinger equation. These solutions have an isomonodromic character, satisfying ordinary differential equations of Painlevé type with respect to the individual independent variables. These results were obtained in several recent papers written jointly with different subsets of the listed co-authors.

TS01-3-2

Applications of a commutation formula II

Percy Deift[1], Fritz Gesztesy[2]

[1]New York University, New York, USA
[2]Baylor University, Waco, USA

Email: deift@cims.nyu.edu

In 1978 one of the authors (PD) wrote a paper on the applications of a commutation formula for operators A and B in a Banach space, to a wide variety of problems in mathematics and physics. Over the years many new applications of the formula to varied, and unanticipated, fields of mathematics have been found. The goal of this talk is to describe some of these new applications.

TS01-4: Day 3, 10:30–12:00, Hardy Hall (Chair: Peter D. Miller)

TS01-4-1

Discrete Painlevé equations from orthogonal polynomials: non-conjugate translations, nodal curves and exotic symmetry groups

Alexander Stokes

Waseda University, Tokyo, Japan

Email: stokes@aoni.waseda.jp

Discrete Painlevé equations are well-known to appear in various contexts related to semi-classical orthogonal polynomials, e.g. via recurrence coefficients, ladder operators or Riemann-Hilbert theory. In our modern understanding of them, discrete Painlevé equations are discrete dynamical systems generated by the actions of extended affine Weyl groups on families of the rational surfaces that were classified by Sakai, and it is natural to ask how different weights match up with different surface types in the Sakai list.

However, the Sakai classification of discrete Painlevé equations is complete only on the level of surfaces and on the level of their extended affine Weyl groups of symmetries in the generic case. Firstly, there can be inequivalent discrete Painlevé equations on the same surface, generated by non-conjugate symmetries. Secondly, equations with parameter constraints live on subfamilies of surfaces, which causes their symmetry groups to be restricted. We show that such examples arise from various generalisations of Laguerre and Meixner orthogonal polynomials, and argue that any correspondence between different weights and the Sakai classification should make use of the refined version of the discrete Painlevé equivalence problem, which takes into account not just surface type, but also the group elements generating the dynamics as well as parameter constraints, e.g. those corresponding to surfaces carrying nodal curves.

This talk is based on joint work with Anton Dzhamay and Galina Filipuk, as well as with Anton Dzhamay, Yang Shi and Ralph Willox.

TS01-4-2

Discrete Painlevé equations with constraints: their geometry and symmetry

Anton Dzhamay

Beijing Institute of Mathematical Sciences and Applications (BIMSA)

Email: adzham@bimsa.cn

Discrete Painlevé equations are discrete dynamical systems on families of Sakai surfaces that admit actions of extended affine Weyl groups. The Painlevé dynamics is generated by the actions of translations or quasi-translations. The Sakai classification of discrete Painlevé equations is complete on the level of surfaces, as well as on the level of the symmetry groups that generate the dynamics in the generic case. However, there are non-generic cases, i.e. discrete Painlevé equations with constraints, in which the true symmetry group is a proper subgroup of the generic one for its type. We consider a class of examples where constraints correspond to, algebraically, setwise stabilizers of some subset of simple roots, and geometrically, to existence of configurations of nodal curves. Such stabilizers can be described explicitly in terms of generators and relations following the techniques developed by B. Brink and R. Howlett. In this way we can obtain discrete Painlevé equations with non-simply laced affine Weyl symmetry groups, as we illustrate by considering some examples on the $\left(A_0^{(1)}\right)^{**}$ Sakai surface family with the $W\left(E_8^{(1)}\right)$ generic symmetry group. This is joint work with Y. Shi, A. Stokes, and R. Willox.

TS01-4-3

Padé method and hypergeometric structures in discrete Painlevé-type systems

Hidehito Nagao

Gifu Shotoku Gakuen University, Gifu, Japan

Email: hnagao@gifu.shotoku.ac.jp

We discuss discrete Painlevé-type systems from the viewpoint of Padé approximation and Padé interpolation methods. A notable feature of the Padé method is that it provides a unified framework for constructing linear problems, deformation/evolution equations, and hypergeometric-type special solutions simultaneously.

Starting from suitable Padé approximation/interpolation problems, we derive scalar Lax equations and the associated compatibility conditions for discrete isomonodromic systems. We also explain how the corresponding special solutions are expressed explicitly in terms of terminating hypergeometric-type functions and determinant formulae. This construction naturally reveals the role of special functions in the structure of discrete integrable systems.

We further discuss possible extensions to higher-dimensional systems, including discrete Garnier systems, and relations with determinant and orthogonality structures. The aim is to present the Padé framework as an effective bridge between special function theory and discrete integrable systems.

TS02: Random Matrices and Probability

Coordinator: Thomas Bothner

TS02-1: Day 4, 10:30–12:00, Hardy Hall (Chair: Karl Liechty)

TS02-1-1

Local universality of determinantal point processes on $\mathbb{C}^d$

Leslie Molag

Carlos III University Madrid, Madrid, Spain

Email: lmolag@math.uc3m.es

The eigenvalues of random normal matrices are known to form a determinantal point process, where the corresponding correlation kernel has the interpretation of a polynomial Bergman kernel depending on a potential. Using polynomial Bergman kernels on $\mathbb{C}^d$, constructed with multivariate complex orthogonal polynomials, these determinantal point processes can be extended to higher complex dimension $d > 1$. We show that the local edge universality observed for $d = 1$, involving the so-called error-function kernel, extends to $d > 1$. Furthermore, we show that, after proper rescaling and rotation, there is a novel universal kernel, which is a multivariate generalization of the error-function kernel.

TS02-1-2

A direct approach to soft and hard edge universality for random normal matrices

Joakim Cronvall

KU Leuven, Belgium

Email: joakim.cronvall@kuleuven.be

The random normal matrix model is a two-dimensional point process describing log-correlated particles in a confining field. The statistics are described by the reproducing kernel of a weighted polynomial Bergman space. Typically, one studies the kernel using asymptotics of orthogonal polynomials. In many interesting situations, such as when the eigenvalues are confined by a hard wall or when the eigenvalues concentrate on disconnected sets, this has turned out to be complicated. In this talk, I will discuss an alternative approach that altogether avoids orthogonal polynomials. Instead we work with Hilbert spaces of entire functions using Paley-Wiener type theorems and potential theory. This approach gives several new universality results and shows in particular how the microscopic behavior of the kernel depends on the local potential theory.

Based on joint work with Aron Wennman

TS02-1-3

Birth of a gap: critical phenomena in the normal matrix model

Sampad Lahiry, Matthias Allard

KU Leuven and University of Melbourne

Email: sampad.lahiry@kuleuven.be

We investigate a family of radially symmetric Coulomb gas systems at inverse temperature $\beta = 2$ coming from the normal matrix model. The family is characterised by the property that the density of the equilibrium measure vanishes on a ring at radius r_*, which lies strictly inside the droplet. The large n expansion of the free energy is obtained up to a novel $n^{1/4}$ term. Next, we perform a scaling limit of the correlation kernel at the $n^{1/4}$ scale and obtain a new limiting kernel in the bulk, which differs from the well-known Ginibre kernel.

Joint work based on arXiv:2509.24529.

TS02-2: Day 4, 15:00–16:30, Hardy Hall (Chair: Lu Wei)

TS02-2-1

On the eigenvalue rigidity of the Jacobi unitary ensemble

Dan Dai, Chenhao Lu

City University of Hong Kong, Kowloon, Hong Kong

Email: chenhaolu3-c@my.cityu.edu.hk

In this paper, we prove an optimal global rigidity estimate for the eigenvalues of the Jacobi unitary ensemble. Our approach begins by constructing a random measure defined through the eigenvalue counting function. We then prove its convergence to a Gaussian multiplicative chaos measure, which leads to the desired rigidity result. To establish this convergence, we apply a sufficient condition from Claeys et al. [1] and conduct an asymptotic analysis of the related exponential moments.

Besides, an ongoing work with Dr. Xiaolu Yue, investigates the similar problem for Laguerre unitary ensemble adopting the similar methods, which will be updated in arXiv soon.

References

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TS02-2-2

Hermitian Jacobi polynomials on the simplex

Teije Kuijper

Aarhus University, Aarhus, Denmark

Email: t.kuijper@math.au.dk

We introduce squared radial coordinates on the partial flag manifolds with blocks of equal size. We then determine eigenpolynomials of the radial part of the Laplace–Beltrami operator. These eigenpolynomials are orthogonal with respect to the complex matrix variate Dirichlet distribution and are called the Hermitian Jacobi polynomials on the simplex. This work is motivated by the study of submatrices of the unitary Brownian motion, and in particular of the simultaneous radial and angular dynamics of the determinants of these submatrices.

This talks starts with a brief overview of these results regarding the unitary Brownian motion. Subsequently, a general introduction to the theory of orthogonal polynomials in the coefficients of a Hermitian matrix invariant under conjugation with unitary matrices and its multivariate extension. In particular a matrix version of Koornwinders method for constructing orthogonal polynomials in multiple variables from orthogonal polynomials in one variable will be given. This is then used to find an explicit expression of the Hermitian Jacobi polynomials on the simplex.

TS02-2-3

On the asymptotic distribution of the largest eigenvalue of unbalanced complex Jacobi ensembles

John J. Lopez Santander[1], Katerina Gkogkou[2], Gustavo Didier[2], Kenneth McLaughlin[2]

[1]University of Manitoba, Winnipeg, Canada
[2]Tulane University, New Orleans, United States

Email: John.LopezSantander@umanitoba.ca

The Jacobi ensemble arises in statistical mechanics as a model for a so-called *log (Coulomb) gas* of charged particles confined to the interval $[0,1]$ and interacting via logarithmic repulsion. The *β-Jacobi ensemble* is the family of probability density functions $f_p(\boldsymbol{\theta}) = \mathcal{Z}_{p,\beta} \cdot \prod_{i=1}^{p}(1-\theta_i)^{c_1\beta/2}\theta_i^{c_2\beta/2}\prod_{i<j}|\theta_i-\theta_j|^{\beta}$, where $\beta > 0$, $c_1, c_2 > -2/\beta$, and $\mathcal{Z}_{p,\beta}$ is a normalization constant.

For a particular choice of the parameters, it also appears in multivariate statistics as the joint eigenvalue distribution of the matrix $J = (A+B)^{-1}B$, where $A = X^*X$ and $B = Y^*Y$, with X and Y being $m \times p$ and $n \times p$ $(m, n \geq p)$ random matrices, respectively, having i.i.d. standard Gaussian entries. The distribution of the largest eigenvalue has been characterized in terms of the Tracy–Widom distribution in the case that m and n grow proportionally with p, for multiple values of β.

This talk presents recent work on the unbalanced regime where m grows faster than linearly with p, while n grows still proportionally to p, for the case $\beta = 2$. In contrast to the case where parameters grow proportionally, where the support of the limiting spectral measure remains fixed, in our case the spectral measure exhibits the distinctive feature that its support shrinks, with both endpoints approaching zero. We prove that after a suitable rescaling, the distribution of the largest eigenvalue is still governed by the Tracy–Widom law and establish a lower-order correction term.

TS02-3: Day 4, 16:45–18:15, Hardy Hall (Chair: Leslie Molag)

TS02-3-1

Dunkl and Bessel processes with drift: motivation, an application to noncompact free Brownian motions, and some limit results

Michael Voit

Technical University Dortmund, Dortmund, Germany

Email: michael.voit@math.tu-dortmund.de

Dyson Brownian motions, i.e., Bessel processes of type A, appear for some parameters as projections of Brownian motions without drift on the vector spaces of Hermitian matrices over $\mathbb{R}, \mathbb{C}, \mathbb{H}$. Moreover, the projections of Brownian motions with drift on these spaces are also diffusions with explicit generators and transition probabilities which can be expressed in terms of Bessel functions of type A. This observation for particular drifts and an identity of Weyl for the spherical functions of $GL(N,\mathbb{C})/U(N,\mathbb{C})$ can be used, for instance, in order to determine the distributions of noncompact free multiplicative Brownian motions of Biane.

Moreover, motivated by these projections of Brownian motions with drift on Weyl chambers of type A, we introduce the notion of Dunkl and Bessel processes with drift for arbitrary root systems and multiplicities $k \geq 0$. These processes have some nice features. In particular, limits for Dunkl kernels and Bessel functions lead to limit results for these processes.

References

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TS02-3-2

Operator norm bounds for multi-leg matrix tensors and applications to random matrix theory

Benoît Collins[1], Wangjun Yuan[2]

[1]Kyoto University, Japan
[2]Southern University of Science and Technology, China

Email: ywangjun@connect.hku.hk

We investigate the extremal values of partial traces of matrix tensors under operator norm constraints. To evaluate these multi-linear quantities, we develop a comprehensive graphical formalism that encodes multi-leg partial traces, partial permutations, and their moments using colored directed graphs. With this graphical framework, we establish optimal, sharp bounds for the partial trace $(\mathrm{Tr}_{\sigma_1} \otimes \ldots \otimes \mathrm{Tr}_{\sigma_k})(A_1, \ldots, A_m)$ over matrices bounded by $\|A_i\| \leq 1$. Specifically, we prove that this maximum evaluates exactly to $N^{M(\sigma_1,\ldots,\sigma_k)}$, where N is the dimension and M represents the maximal number of directed cycles in the associated graph across all possible internal vertex pairings. We further derive explicit operator norm estimates for matrices generated by partial traces of partial permutations. Finally, we apply these combinatorial bounds to multi-matrix random matrix theory. By examining models involving Ginibre ensembles, we extend concepts of asymptotic freeness to matrix coefficient algebras, establishing operator norm estimates that rigorously separate the asymptotic behavior of non-crossing and crossing pairings.

TS02-3-3

Dissipative spectral form factor of the complex elliptic Ginibre ensemble across various non-Hermiticity regimes

Gernot Akemann[1], Sung-Soo Byun[2], Seungjoon Oh[2]

[1]Bielefeld University, Bielefeld, Germany
[2]Seoul National University, Seoul, Republic of Korea

Email: seungjoonoh@snu.ac.kr

We study the dissipative spectral form factor (DSFF) at complex time $Te^{i\theta}$ for the complex elliptic Ginibre ensemble with non-Hermiticity parameter $\tau \in [0, 1)$. As the matrix dimension $N \to \infty$, we consider the natural scalings in both the time variable and the non-Hermiticity parameter, namely $T = O(N^{\gamma})$ and $1-\tau = O(N^{-\alpha})$. For all regimes $\gamma \geq 0$ and $\alpha \geq 0$, we derive the precise asymptotic behaviour of both the disconnected and connected components of the DSFF. In particular, we explicitly characterise the dip–ramp–plateau structure, including the dip time and the Heisenberg time. In addition, we identify the mesoscopic regime $\alpha \in (0, 1)$, which interpolates between the DSFF behaviour of non-Hermitian random matrices and the spectral form factor (SFF) behaviour of Hermitian ensembles. We further provide an explicit description of the phase diagram, in which the ramp exhibits quadratic, linear, or intermediate behaviour depending on the scaling parameters.

TS02-4: Day 5, 10:30–12:00, Hardy Hall (Chair: Johan Kuijper)

TS02-4-1

Deformations of OP Ensembles at a regular bulk point

Caio Candido[1], Victor Alves[2], Thomas Chouteau[1], Charles F. Santos[3], Guilherme L. F. Silva[1]

[1]Universidade de São Paulo, São Carlos, Brazil
[2]Fudan University, Shanghai, China
[3]Universidade Federal de Minas Gerais, Belo Horizonte, Brazil

Email: victoralves@fudan.edu.cn

We consider orthogonal polynomial ensembles with weights given by deformations of exponential weights, analyzing the regime where the number of particles is large. The deformation symbols under study influence local fluctuations near a bulk point of the limiting equilibrium measure. We show that the limiting correlation kernel can be expressed via a solution to an integrable non-local differential equation. This new kernel can be identified as the correlation kernel of a conditionally thinned process derived from the Sine point process, and it also connects to a finite-temperature deformation of the Sine kernel recently examined by Claeys and Tarricone. In addition, we investigate how the deformation impacts the recurrence coefficients of the associated orthogonal polynomials; these coefficients exhibit oscillatory behavior even when the limiting equilibrium measure corresponds to a regular one-cut scenario.

TS02-4-2

q-deformation of the Marchenko-Pastur law

Sung-Soo Byun[1], Yeong-Gwang Jung[1], Guido Mazzuca[2]

[1]Seoul National University, Seoul, Korea
[2]Tulane University, New Orleans, USA

Email: wollow21@snu.ac.kr

In this talk, I will present a q-deformed random unitary ensemble associated with the little-q Laguerre weight, which provides a discrete analogue of the Laguerre unitary ensemble. In the double scaling regime $q = e^{-\lambda/N}$ where N is the system size and λ is an additional parameter, the limiting spectral distribution yields the q-deformation of the Marchenko-Pastur law. The key feature of the limiting density is the presence of a phase transition depending on the value of λ, due to an upper constraint. I will discuss three complementary approaches to the limiting density: the method of moments, the analysis of a constrained equilibrium problem, and the asymptotic zero distribution of q-orthogonal polynomials. In addition, I will introduce the underlying combinatorial structure of the spectral moments.

This talk is based on joint work with Sung-Soo Byun and Guido Mazzuca.

TS02-4-3

Average entropy of Bogoliubov-Kubo-Mori random state ensemble

Sohail[1], Lu Wei[2]

[1]Chapman University, USA
[2]Texas Tech University, USA

For an $m \times m$ random density matrix ρ, it was recently found by Harry Miller (arXiv: 2511.01988) that the joint eigenvalue density of ρ,

$$f(\lambda) = \frac{1}{C}\,\delta\left(1 - \sum_{i=1}^{m} \lambda_i\right) \prod_{1 \le i < j \le m} (\lambda_i - \lambda_j)(\ln \lambda_i - \ln \lambda_j) \prod_{i=1}^{m} \lambda_i^{\alpha}, \qquad 0 < \lambda_i < 1, \tag{1}$$

is the most natural random state ensemble induced from von Neumann entropy $S = -\sum_{i=1}^{m} \lambda_i \ln \lambda_i$ under the Bogoliubov-Kubo-Mori (BKM) distance metric. In the special case $\alpha = -1/2$, the limiting average entropy was also computed, based on the connection to Muttalib-Borodin ensemble, by Miller as

$$\mathbb{E}_f[S] = \ln m + \psi_0(1) - \ln 2 + \frac{1}{2} + O\left(\frac{\ln m}{m}\right). \tag{2}$$

The main result of this work is an exact yet explicit average entropy formula of the BKM ensemble as

$$\mathbb{E}_f[S] = \psi_0(\beta + 1) - \frac{1}{\beta}\left(\sum_{i=0}^{m-1} \binom{m}{i+1} \frac{1}{i!}\left(\alpha + \frac{m+1}{i+2}\right)\psi_i(\alpha + 1) + \frac{1}{2}m(m+1)\right), \tag{3}$$

where $\beta = m(\alpha + \frac{m+1}{2})$ and ψ_i is the i-th polygamma function. In obtaining (3), one only makes use of the ensemble density (1) in the absence of its correlation kernel and orthogonal polynomials. As a useful property in quantum information processing, the BKM ensemble is shown in Miller's work to attain minimal average entropy among other major generic state ensembles in the asymptotic regime. The exact formula (3) confirms the validity of this property for arbitrary system dimensions.

TS02-5: Day 5, 15:00–16:30, Hardy Hall (Chair: Joakim Cronvall)

TS02-5-1

Multiple orthogonal polynomials appearing in the six-vertex model with symmetries

Karl Liechty[1], Guilherme Silva[2], Jimmy He[3], Vadim Gorin[4]

[1]DePaul University, Chicago, USA
[2]University of São Paulo, Brazil
[3]The Ohio State University, USA
[4]University of California, Berkeley, USA

Email: `kliechty@depaul.edu`

I will discuss the analysis, via Riemann–Hilbert problem, of some mixed-type multiple orthogonal polynomials arising in the study of the six-vertex model of statistical mechanics. These polynomials have the following property which may be of independent interest: the odd part of the polynomial times a weight satisfies its own orthogonality condition.

TS02-5-2

Free energy expansion of determinantal Coulomb gases in the quadratic fields with a point charge

Sung-Soo Byun[1], Meng Yang[2], Eui Yoo[1]

[1]Seoul National University, Seoul, Republic of Korea
[2]Central South University, Changsha, China

Email: yysh0227@snu.ac.kr

In this talk, we will discuss the free energy expansion of non-Hermitian random matrices, in view of determinantal Coulomb gases. To be specific, we focus on a determinantal Coulomb gas model in the complex plane associated with an external potential

$$Q(z) = \frac{1}{1-\tau^2}(|z|^2 - \tau \mathrm{Re}\, z^2) - 2c \log |z-a|,$$

where $\tau \in [0,1)$, $c \geq 0$ and $a \geq 0$. In the regimes where the associated droplet is either simply or doubly connected, we derive the free energy expansion up to and including the constant term, with all coefficients given explicitly.

For the proof, we present a robust deformation framework involving both the location of the singularity a and the anisotropy parameter τ, relating the variations of the free energy to fine asymptotics of planar orthogonal polynomials. The asymptotic analysis is then based on the foliation flow method of Hedenmalm and Wennman, offering an alternative to the Riemann–Hilbert approach used in previous results. Under this framework, the topological and conformal geometric data of the Coulomb gas model systematically relate to the fine asymptotics of planar orthogonal polynomials in terms of the Liouville action.

TS02-5-3

Asymptotics of the partition function with a one-cut singular potential at the edge

Dan Dai, Jiahao Du, Chenhao Lu

City university of Hong Kong, Kowloon, Hong Kong

We study the large n asymptotics of the partition function for a Hermitian random matrix model with an even analytic potential V whose equilibrium measure is supported on $[-A, A]$ and has a $5/2$-order critical behavior at the endpoints. In contrast with the one-cut regular case, where the Airy parametrix gives the local behavior at the soft edge, the present critical regime requires a higher-order local model related to the second member of the Painlevé I hierarchy. In particular, in the double scaling limit, the Hamiltonian corresponding to the P_I^2 equation appears explicitly in the constant term of the partition function asymptotics. This is a joint work in progress with Dan Dai and Chen-Hao Lu.

TS03: Exact WKB Analysis and Related Topics
(Topological Recursion, Conformal Blocks, Beta-Ensembles)

Coordinator: Andrei Prokhorov

TS03: Day 2, 17:15–18:15, KMB 212 (Chair: Yoshitsugu Takei)

(continuing from TS13-3 16:45–17:15)

TS03-1-1

Small-time and large-time Borel analysis of Schrödinger equations with point interactions

Ovidiu Costin[1], Gerald V. Dunne[2], Bart Rosenzweig[1], Jonathan Stanfill[2]

[1]The Ohio State University, Columbus, USA
[2]University of Connecticut, Storrs, USA

Email: rosenzweig.26@osu.edu

In this talk we will discuss our recent proofs of Borel summability of the $t \to 0^+$ and $t \to \infty$ asymptotic expansions of the solutions to the time-dependent Schrödinger equation with generalized point interactions. As was observed for the heat kernel on hyperbolic spaces in [3], the Borel singularity structures are markedly different in either of these asymptotic regimes. The large-time analysis builds on previous work of [1] and involves careful asymptotics of the resonances in the complex plane. The small-time analysis offers an alternative perspective to that used to study the heat equation in [2], and involves the analytic continuation in time of the wave propagator $\cos(t\sqrt{H})$.

References

[1] O. Costin and M. Huang, *Gamow Vectors and Borel Summability in a Class of Quantum Systems*, J. Stat. Phys. 144 (2011).

[2] O. Costin, H. Park, and Y. Takei, *Borel summability of the heat equation with variable coefficients*, J. Differential Equations 252, 4 (2012).

[3] G.V. Dunne, "Borel Summation and Analytic Continuation of the Heat Kernel on Hyperbolic Space" in *Peter Suranyi 87th Birthday Festschrift*, World Scientific, Singapore, 2022.

TS03-1-2

Exact resurgence in exact WKB analysis

Gergő Nemes

Harbin Institute of Technology, Harbin, China

Email: g.nemes@hit.edu.cn

This talk presents recent progress on explicit remainder representations for WKB expansions in the form of exact resurgence formulae. These formulae provide global representations of Borel-resummed WKB expansions, naturally encode the Stokes phenomenon, and yield computable and realistic error bounds. They furthermore provide the basis for hyperasymptotic methods, allowing the systematic incorporation of exponentially small corrections and leading to highly accurate numerical approximations. Applications include the explanation of higher-order Stokes phenomena, such as the so-called "ghost-like" effect.

TS04: Orthogonal Polynomials, Mathematical Physics, and Interacting Particle Systems

Coordinator: Sarah Post

TS04-1: Day 5, 10:30–12:00, KMB 212 (Chair: Sarah Post)

TS04-1-1

The dynamical algebra of the generic superintegrable model on the two-sphere

Nicolas Crampé[1,2], Quentin Labriet[3], Lucia Morey[3], Satoshi Tsujimoto[4], Luc Vinet[3], Alexei Zhedanov[5]

[1]CNRS - Université de Tours, Tours, France
[2]Laboratoire d'Annecy de Physique Théorique, Annecy, France
[3]Université de Montréal, Montréal, Canada
[4]Kyoto University, Kyoto, Japan
[5]Leonhard Euler International Mathematical Institute, Saint Petersburg, Russian Federation

Email: luc.vinet@umontreal.ca

The rank two Jacobi algebra $\mathfrak{J}_2$ is identified as the dynamical algebra of the generic quadratic superintegrable model on the two-sphere. The physical representation of this algebra is obtained from its embedding in $\mathfrak{su}(1,1)^{\otimes 3}$. The exact solution of the model is derived algebraically from this representation. The wavefunctions are found to be expressed in terms of two-variable Jacobi polynomials whose characterization is a by-product of the algebraic treatment of the model.

TS04-1-2

Symmetry operators and Stäckel separation of Dirac's equation on 3-dimensional spin manifolds

L. Fatibene[1], R. G. McLenaghan[2], G. Rastelli[1]

[1]University of Turin, Turin, Italy
[2]University of Waterloo, Waterloo, Canada

Email: lorenzo.fatibene@unito.it, rgmclena@uwaterloo.ca, giovanni.rastelli@unito.it

We consider the Dirac equation on three-dimensional spin manifolds. The unknowns of the equation are naturally two-spinors and we investigate the integration of the equation by separation of variables by assuming that each component of the spinor is multiplicatively separated. Our approach differs from previous ones since we consider separation of the square of the Dirac equation instead of the Dirac equation itself. We show that this approach allows in a natural way the application of Stäckel's theory of separation, developed for second-order differential equations of Schrödinger type. We determine the second-order symmetry operators of the Dirac equation, and therefore of the Dirac squared equation, directly from the Killing tensors associated with Stäckel separation, and show how they may be used to characterize separation of varaiables for the Dirac squared equation. The separated solutions are therefore determined by two uncoupled equations of Schrödinger type. The separated solutions obtained in this way are also separated solutions of the Dirac equation, provided straightforward conditions on the integration constants are satisfied. We show how all the known cases of Dirac separated solutions in Stäckel coordinates of 3-dimensional Euclidean space can be systematically recovered by our approach after a suitable choice of the spin frame. As a consequence, the characterization of the separated solutions of the Scrödinger equations in terms of orthogonal polynomials extends to the separated solutions of the Dirac equation.

TS04-1-3

Two-dimensional Coulomb gases with rotation-invariant potentials with singularities

Kohei Noda

Université catholique de Louvain, Louvain-la-Neuve, Belgium

Email: kohei.noda@uclouvain.be

For $z_n = (z_1, \dots, z_n) \in \mathbb{C}^n$, we consider the n-fold integral

$$D_n(Q, f, \vec{s}_U, U) := \int_{\mathbb{C}^n} \prod_{1\le i<j\le n} |z_i - z_j|^2 \prod_{j=1}^{n} e^{-nQ_U(z_j)} \omega(z_j)\, dA(z_j),$$

where $dA(z) = \frac{dx\,dy}{\pi}$ denotes the Lebesgue measure on the complex plane $\mathbb{C}$. The external potential Q_U is defined by $Q_U(z) := Q(z)\mathbf{1}_{\mathbb{C}\setminus U}(z) + \infty\,\mathbf{1}_U(z)$, where $Q(z) = q(|z|)$ is a smooth rotation-invariant potential. The set U is taken to be one of the following: (i) a centered disk, (ii) the complement of a centered disk, and (iii) a centered annulus, contained in the droplet. The function $\omega(z)$ consists of a smooth rotation-invariant test function together with jump-type singularities. In this setting, we establish the large-n asymptotic expansion of $D_n(Q, f, \vec{s}_U, U)$. In this talk, I will unify and extend several asymptotic formulas obtained in a series of works by Ameur, Charlier, Cronvall, and Lenells. The resulting large-n expansion of $D_n(Q, f, \vec{s}_U, U)$ has several applications, including the analysis of hole probabilities, the multivariate moment generating functions of disk counting statistics, and the moment generating functions of smooth linear statistics.

TS04-2: Day 5, 15:00–16:00, KMB 212 (Chair: Hiroshi Miki)

TS04-2-1

Integral equations for Sheffer-Bessel functions through umbral operator

Umme Zainab[1], Manoj Kumar[2]

[1]Jamia Millia Islamia, New Delhi, India
[2]SR University, Warangal, India

Email: umme.zainab15@gmail.com, manojrajpot066@gmail.com

In this article, we explore the theoretical foundations of hybrid families of special functions through an umbral reformulation, focusing specifically on Sheffer-Bessel functions. Our discussion integrates various families of Bessel-type functions and other special polynomials within a cohesive umbral-algebraic framework. This approach leverages recent advancements over the formal treatment of higher transcendental functions, facilitating novel and intriguing generalisations. By applying this method to Sheffer-Bessel functions, we aim to provide new insights and extend the understanding of these mathematical entities.

TS04-2-2

Exceptional orthogonal polynomials, exactly solvable quantum systems and their deformations

Siyu Li[1], Ian Marquette[2], Sarah Post[3,4], Yao-Zhong Zhang[4]

[1]La Trobe University, Bundoora, Australia
[2]La Trobe University, Bendigo, Australia
[3]University of Hawaii, Honolulu, USA
[4]The University of Queensland, Brisbane, Australia

Email: spost@hawaii.edu

In this talk, I will briefly review the connection between exactly solvable quantum systems and exceptional orthogonal polynomials (EOPs). In particular, the EOPs and their associated Darboux transformation offer a mechanism to construct vast families of exactly-solvable and integrable systems with non-contiguous energy levels. Just as with other exactly solvable systems, we can construct quasi-exactly solvable deformations, which will be discussed at the end.

TS05: Algebraic Combinatorics

Coordinator: Hajime Tanaka

TS05-1: Day 3, 10:30–12:00, KMB 213 (Chair: Hajime Tanaka)

TS05-1-1

Strengthening the balanced set condition for the distance-regular graph of the bilinear forms

Paul Terwilliger[1], Jason Williford[2]

[1]University of Wisconsin-Madison, USA
[2]University of Wyoming, USA

Email: terwilli@math.wisc.edu, jwillif1@uwyo.edu

We consider a distance-regular graph $\Gamma = (X, \mathcal{R})$ called the bilinear forms graph $H_q(D, N-D)$; we assume $N > 2D \geq 6$ and $q \neq 2$. We show that Γ satisfies the following strengthened version of the balanced set condition. For a vertex $x \in X$ and $0 \leq i \leq D$ define $\Gamma_i(x) = \{y \in X | \partial(x,y) = i\}$, where ∂ denotes the path-length distance function. Abbreviate $\Gamma(x) = \Gamma_1(x)$. Let $V = \mathbb{R}^X$ denote the standard module for $\mathrm{Mat}_X(\mathbb{R})$. For $x \in X$ let $\hat{x} \in V$ have x-coordinate 1 and all other coordinates 0. Let $E \in \mathrm{Mat}_X(\mathbb{R})$ denote the primitive idempotent that corresponds to the second largest eigenvalue of the adjacency matrix of Γ. For a subset $\Omega \subseteq X$ define $\widehat{\Omega} = \sum_{x\in\Omega} \hat{x}$. We fix two vertices $x, y \in X$ and write $k = \partial(x,y)$. To avoid degenerate situations, we assume $2 \leq k \leq D-1$. Using y we obtain an equitable partition $\{O_i\}_{i=1}^6$ of the local graph $\Gamma(x)$. By construction $O_1 = \Gamma(x) \cap \Gamma_{k-1}(y)$ and $O_6 = \Gamma(x) \cap \Gamma_{k+1}(y)$. We call $\{O_i\}_{i=1}^6$ the y-partition of $\Gamma(x)$. Let $\{O'_i\}_{i=1}^6$ denote the x-partition of $\Gamma(y)$. According to the original balanced set condition, for $i \in \{1,6\}$ the vector $E\widehat{O}_i - E\widehat{O}'_i$ is a scalar multiple of $E\hat{x} - E\hat{y}$. We show that for $1 \leq i \leq 6$ the vector $E\widehat{O}_i - E\widehat{O}'_i$ is a scalar multiple of $E\hat{x} - E\hat{y}$. We investigate the consequences of this result.

TS05-1-2

Imprimitive association schemes and elimination theory

Hirotake Kurihara[1], Akihiro Higashitani[2]

[1]Yamaguchi University, Ube, Japan
[2]Osaka University, Suita, Japan

Email: kurihara-hiro@yamaguchi-u.ac.jp

In this talk, we establish a fundamental characterization of imprimitivity in commutative association schemes through the lens of **multivariate P- or Q-polynomial structures**. We prove that a scheme is imprimitive if and only if it admits such a structure with respect to an elimination-type monomial order. This result provides a direct bridge between the classical combinatorial theory of block and quotient schemes and the framework of elimination theory in computational commutative algebra.

For an imprimitive multivariate P- or Q-polynomial association scheme, we explicitly determine the induced polynomial structures on its quotient and block schemes. These are described via specific transformations of the original associated polynomials. At the algebraic level, we demonstrate that the ideal of the block scheme is precisely an **elimination ideal**, while the ideal of the quotient scheme arises from adjoining valency relations and subsequent elimination.

Furthermore, we apply this theory to characterize direct products and crested products. In particular, we show that schemes which remain multivariate P- or Q-polynomial under every monomial order are exactly the direct products of univariate P- or Q-polynomial schemes. We conclude by discussing implications for formal duality, composition series, and several promising open problems in the field.

TS05-1-3

Irreducible representations of the S_3-symmetric tridiagonal algebra: the complete graph case

Hajime Tanaka, Yushan Pan

Tohoku University, Sendai, Japan

Email: htanaka@tohoku.ac.jp, pan.yushan.t4@dc.tohoku.ac.jp

The tridiagonal algebra T is defined by two generators and two relations known as the tridiagonal relations, which encompass several important algebraic structures such as the q-Onsager algebra and the Onsager Lie algebra. In this talk, we focus on a recently introduced generalization termed the S_3-symmetric tridiagonal algebra $\mathbb{T}$. This algebra is characterized by six generators $\{A_i, A_i^*\}_{i=1}^3$ identified with the vertices of a regular hexagon, where nonadjacent generators commute and adjacent ones satisfy the tridiagonal relations.

We turn the tensor power $V^{\otimes 3}$ of the standard module V of the complete graph K_N on N vertices into a module of $\mathbb{T}$. Our primary contribution is an explicit construction of an irreducible decomposition of this module.

We provide:

1. An irreducible decomposition of the $\mathbb{T}$-module $V^{\otimes 3}$;
2. A description of an eigenbasis of the Jucys-Murphy elements in $\mathbb{C}[S_N]$ within these irreducible components.

This is a joint work with Prof. Hajime Tanaka.

TS05-2: Day 4, 10:30–12:00, KMB 213 (Chair: Hajime Tanaka)

TS05-2-1

Hausdorff construction and combinatorial polynomials

Hideki Matsumura[1], Kosuke Minami[2], Teruyuki Mishima[3] and Masanori Sawa[2]

[1]Tokyo Metropolitan University, Tokyo, Japan
[2]Kobe University, Kobe, Japan
[3]Accenture, Osaka, Japan

Email: hmatsumura@tmu.ac.jp

In classical Waring's problem in additive number theory, Hilbert solved the problem using an algebraic identity (Hilbert identity), which is known to be equivalent to certain spherical designs. His original proof, however, was long and complicated. Hausdorff simplified Hilbert's argument by using Gauss–Hermite quadrature. In this talk, we apply the Hausdorff construction to moments of classical orthogonal polynomials other than Hermite polynomials, including Laguerre, Bessel and Chebyshev polynomials, and derive explicit polynomial identities. Furthermore, when specialized to two variables, the resulting identities exhibit connections to classical combinatorial polynomials such as Narayana polynomials. We also provide an alternative proof of the Hausdorff lemma via convex algebraic geometry.

TS05-2-2

Sharp bounds for equi-chordal tight fusion frames and Grassmann designs

Ryutaro Misawa

Tohoku University, Sendai, Japan

Reproducing the average properties of a continuous space by finitely many points or subspaces is a basic problem in design theory. In this talk, we study Grassmann designs and tight fusion frames on the real Grassmannian $G_{k,d}$, the space of all k-dimensional subspaces of $\mathbb{R}^d$, and investigate their structure and cardinality bounds by using orthogonal polynomials.

Just as spherical designs can be characterized by Gegenbauer polynomials, Grassmann designs can be characterized by zonal polynomials on the Grassmannian. Using this polynomial characterization, we establish several results on tight Grassmann designs and equi-chordal tight fusion frames. First, we prove that tight Grassmann 2-designs do not exist for any k and d. We also show that, if a tight Grassmann 4-design exists, then it must be equi-chordal. Moreover, for an equi-chordal tight 2-fusion frame $D \subset G_{2,d}$, we prove the sharp cardinality bound

$$\frac{d^2}{4} \leq |D| \leq \binom{d+1}{2}.$$

The lower equality case corresponds to equi-isoclinic tight 2-fusion frames, while the upper equality case corresponds to tight Grassmann 4-designs.

The key ingredient of the proof is the explicit use of low-degree zonal polynomials, which form an orthogonal polynomial system on the Grassmannian. We rewrite the design and tight fusion frame conditions as vanishing conditions for sums of these zonal polynomials, and then translate them into symmetric expressions in the principal angles between pairs of subspaces. This translation, combined with the AM-GM inequality and the chordal embedding of the Grassmannian, yields the nonexistence theorem, the rigidity result, and the sharp cardinality bounds above.

TS05-2-3

Algebraic quantum graph theory and application to orthogonal polynomials

Junichiro Matsuda

Seoul National University, Seoul, Korea

Email: j.mathsuda@gmail,com

Quantum graphs are a non-commutative analogue of graphs with vertices and edges. They were introduced motivated by quantum information theory in the early 2010's, and they have been algebraically investigated since the introduction of their adjacency matrices in 2018 by Musto, Reutter, and Verdon. It is classically known that a sort of association schemes, a good family of adjacency matrices, generates orthogonal polynomials with respect to the eigenvalue distribution of the generating adjacency matrix. I will briefly overview the algebraic approaches to quantum graph theory, and show its potential application to orthogonal polynomials indexed by quantum sets.

TS05-3: Day 4, 15:00–16:30, KMB 213 (Chair: Hajime Tanaka)

TS05-3-1

An explicit formula for Koornwinder moments and Rains' positivity conjecture

Younggwang Cho[1], Donghyun Kim[2], Jang Soo Kim[1], Hojoon Lee[1], Jing Liu[3], Minho Song[4]

[1]Sungkyunkwan University, Suwon, South Korea
[2]Ewha Womans University, Seoul, South Korea
[3]Shandong University, Qingdao, P.R. China
[4]Yonsei University, Seoul, South Korea

Email: brglory@g.skku.edu, kdh310@ewha.ac.kr, jangsookim@skku.edu, hojoon1101@g.skku.edu, lsweet@mail.sdu.edu.cn, smh3227@gmail.com

The Askey–Wilson polynomials are the orthogonal polynomials which lie at the top of the hierarchy of classical orthogonal polynomials. Sasamoto showed that their moments are closely related to partition functions of the asymmetric exclusion process (ASEP), a stochastic particle system on a one-dimensional lattice in which particles hop left and right with open boundaries. It is also known that the Askey–Wilson polynomials are a special case of the Koornwinder polynomials, also known as Macdonald polynomials of type BC. The positivity phenomenon in univariate Koornwinder polynomials is proven using the partition functions of the ASEP. Motivated by this, Rains conjectured that the Koornwinder moments are polynomials with positive coefficients, after a suitable normalization. In our work, we present an explicit formula for Koornwinder moments with specified normalizing factor and prove the conjecture in two special cases: $(\xi, q) = (1, 0)$ and $(\xi, q) = (1, 1)$. For $(\xi, q) = (1, 0)$, we express these moments as a generating functions for families of nonintersecting lattice paths, so that positivity follows from the Lindström–Gessel–Viennot lemma. For $(\xi, q) = (1, 1)$, we derive an explicit product formula and interpret it in terms of lecture hall tableaux.

TS05-3-2

Moments and lattice paths for orthogonal polynomials of type R_{II}

Jang Soo Kim[1], Minho Song[2]

[1]Sungkyunkwan University, Suwon, South Korea
[2]Yonsei University, Seoul, South Korea

Email: minhosong@yonsei.ac.kr

The combinatorial theory of orthogonal polynomials explains moments by weighted lattice path models. For classical orthogonal polynomials this is given by Motzkin paths, and related models are known for Laurent biorthogonal polynomials and for orthogonal polynomials of type R_I.

In this talk, we extend this viewpoint to orthogonal polynomials of type R_{II}, introduced by Ismail and Masson. Unlike the classical and type R_I cases, the type R_{II} setting naturally leads to a linear functional on a space of rational functions. To develop a moment theory in this setting, we enlarge the space and construct a weighted path model for the resulting generalized moments.

We then introduce generalized R_{II} polynomials by relaxing the original conditions and prove a master theorem that unifies the known path models for classical orthogonal polynomials, Laurent biorthogonal polynomials, and orthogonal polynomials of types R_I and R_{II}. We also study dual coefficients. Although type R_{II} polynomials are not monic and their dual coefficients need not be polynomials, we prove that their formal power series expansions have nonnegative integer coefficients by interpreting them using restricted R_{II} paths.

As a final application, we consider the case of constant recurrence coefficients. In this case, the seemingly infinite R_{II}-path sums for the moments can be converted into finite Motzkin and Motzkin–Schroder path sums. This gives a practical way to compute the moments and also shows why the generalized moments require a different model.

TS05-3-3

Discriminants of orthogonal polynomials via symmetric functions

Mourad E.H. Ismail[1], Jihyeug Jang[2], Jang Soo Kim[3], Dennis Stanton[4]

[1]University of Louisiana at Lafayette, Lafayette, USA
[2]Université de Genève, Genève, Switzerland
[3]Sungkyunkwan University, Suwon, South Korea
[4]University of Minnesota, Minneapolis, USA

Email: mourad.eh.ismail@gmail.com, jihyeugjang@gmail.com, jangsookim@skku.edu, stant001@umn.edu

The discriminant of a polynomial is a fundamental symmetric expression in its roots, but it is difficult to compute in general. For many families of orthogonal polynomials, however, their discriminants are given by remarkably simple product formulas. In this ongoing project, we develop a combinatorial method for deriving such formulas.

To compute discriminants in this framework, we introduce a new family of symmetric functions, which we call *generalized power sum symmetric functions*. They are defined from a given orthogonal polynomial together with a suitable difference operator. We then define *companion orthogonal polynomials*, whose moments are given by these generalized power sum symmetric functions. The three-term recurrence coefficients of the companion orthogonal polynomials play the central role: they allow us to derive formulas for the discriminants of the original polynomials. This is joint work with Mourad Ismail, Jang Soo Kim, and Dennis Stanton.

TS05-4: Day 4, 16:45–17:45, KMB 213 (Chair: Hajime Tanaka)

TS05-4-1

Two bases for the Onsager Lie algebra $\mathcal{O}$ and their action on finite-dimensional irreducible $\mathcal{O}$-modules

Jae-Ho Lee

University of North Florida, USA

Email: jaeho.lee@unf.edu

The Onsager Lie algebra $\mathcal{O}$ is infinite-dimensional; it is presented by two generators A, B and two relations called the Dolan-Grady relations. This talk consists of two parts. In the first part, we introduce four types of elements of $\mathcal{O}$, denoted $\{A_k\}_{k\in\mathbb{N}}$, $\{B_k\}_{k\in\mathbb{N}}$, $\{\psi_{k+1}\}_{k\in\mathbb{N}}$, $\{\tilde{\psi}_{k+1}\}_{k\in\mathbb{N}}$. By construction, $A_0 = A$ and $B_0 = B$. We show that $\{A_k\}_{k\in\mathbb{N}}$ mutually commute, $\{B_k\}_{k\in\mathbb{N}}$ mutually commute, $\{\psi_{k+1}\}_{k\in\mathbb{N}}$ mutually commute, and $\{\tilde{\psi}_{k+1}\}_{k\in\mathbb{N}}$ mutually commute. We show that $\{A_k, B_k, \psi_{k+1}\}_{k\in\mathbb{N}}$ forms a basis for $\mathcal{O}$ and $\{A_k, B_k, \tilde{\psi}_{k+1}\}_{k\in\mathbb{N}}$ forms a basis for $\mathcal{O}$. We describe how these two bases are related to each other, and how they are related to the basis $\{W_{-k}, W_{k+1}, \tilde{G}_{k+1}\}_{k\in\mathbb{N}}$ of $\mathcal{O}$ introduced by Baseilhac and Crampé (2019). In the second part, we consider a finite-dimensional irreducible $\mathcal{O}$-module V. It is known that A, B act on V as a tridiagonal pair. Associated with this pair are six decompositions of V; these are the eigenspace decomposition of A, the eigenspace decomposition of B, and four decompositions of V said to be split. We describe how the elements $\{A_k, B_k, \psi_{k+1}, \tilde{\psi}_{k+1}\}_{k\in\mathbb{N}}$ act on these decompositions. We show that for each of these decompositions, the action is block diagonal or block upper bidiagonal or block lower bidiagonal or block tridiagonal. Our work is motivated by some results of Terwilliger (2023) concerning the alternating elements of the positive part U_q^+ of $U_q(\widehat{\mathfrak{sl}}_2)$.

TS05-4-2

Verma modules over the universal Askey–Wilson algebra

Hau-Wen Huang

National Central University, Taoyuan, Taiwan

Email: hauwenh@math.ncu.edu.tw

The Askey–Wilson algebras encode the bispectral properties of orthogonal polynomials in the Askey scheme. In 2001, P. Terwilliger proposed the universal Askey–Wilson algebra $\triangle_q$, depending on a nonzero parameter q, which is a central extension of those Askey–Wilson algebras corresponding to the most general orthogonal polynomials in the scheme.

In this talk, I will introduce the Verma $\triangle_q$-modules, which are a family of infinite-dimensional $\triangle_q$-modules. When q is not a root of unity, every finite-dimensional irreducible $\triangle_q$-module is isomorphic to a quotient of a Verma $\triangle_q$-module. When q is a root of unity, the same holds for every finite-dimensional irreducible $\triangle_q$-module with marginal weights.

TS06: Combinatorics, q-Series, Partitions, and Number Theory

Coordinator: Shuhei Kamioka

TS06-1: Day 5, 10:30–12:00, KMB 213 (Chair: Shuhei Kamioka)

TS06-1-1

Universal rational Q-functions

Soichi Okada

Nagoya University, Nagoya, Japan

Email: okada@math.nagoya-u.ac.jp

In 2009, Ogawa introduced a family of symmetric functions $Q_{[\lambda,\mu]}(X,Y)$ in two sets of infinitely many variables X and Y, indexed by pairs of strict partitions, and proved that they solve a series of non-linear differential equations called the BUC hierarchy. We call $Q_{[\lambda,\mu]}(X,Y)$ the universal rational Q-functions. Universal rational Q-functions are a generalization of Schur's Q-functions, and can be regarded as a Q-function version of universal rational characters, which specialize to the characters of irreducible rational representations of general linear groups.

In this talk, we will discuss several formulas involving universal rational Q-functions, such as the Cauchy-type formula and the Schur-type Pfaffian formula. Also we give a Nimmo-type formula for a specialization $X = (x_1, \dots, x_n, 0, \dots)$ and $Y = (x_1^{-1}, \dots, x_n^{-1}, 0, \dots)$.

TS06-1-2

Multiple polylogarithms, symmetric functions and discrete integrable systems

Masaki Kato

Ritsumeikan University, Osaka, Japan

Email: katom@fc.ritsumei.ac.jp

In this talk, we study a family of special functions arising from elliptic q-multiple polylogarithms and their connections to the algebra of (quasi)symmetric functions and to discrete integrable systems. Motivated by Schur functions and their quasisymmetric generalizations, we define functions $\mathcal{S}^{\gamma}_{\lambda/\mu}$ attached to partitions λ, μ by replacing the underlying quasisymmetric building blocks with suitable elliptic q-multiple polylogarithms. This construction yields p, q-deformations of Schur multiple zeta values through Taylor expansions. Using a ring homomorphism from QSym to an appropriate function ring, we obtain analogues of classical identities for Schur functions, including a Jacobi–Trudi type determinantal formula.

We then explain how this determinantal structure can be exploited to produce discrete integrable dynamics. Combining quadratic identities for determinants with q-difference relations of the underlying elliptic q-multiple polylogarithms, we derive Hirota-type bilinear difference equations satisfied by $\mathcal{S}^{\gamma}_{\lambda/\mu}$. In particular cases, the resulting bilinear system reduces to known discrete integrable systems, including the R_I lattice and the discrete relativistic Toda lattice.

TS06-1-3

Characterization of non-singular hyperplanes of $\mathcal{H}(s, q^2)$ in $\mathrm{PG}(s, q^2)$

Stuti Mohanty, Bikramaditya Sahu

National Institute of Technology Rourkela, Rourkela, India

Email: stutimohanty034@gmail.com

In this paper, we present a combinatorial characterization of the non-singular hyperplanes defined with respect to a non-singular hermitian variety $\mathcal{H}(s, q^2)$ in the projective space $\mathrm{PG}(s, q^2)$ where $s \geq 3$ and $q > 2$. By analyzing the intersection numbers of hyperplanes with points and co-dimension 2 subspaces, we establish necessary and sufficient conditions for a collection of hyperplanes to be the set of all non-singular hyperplanes of a hermitian variety. This approach extends previous characterizations of hermitian varieties based on intersection properties, providing a purely combinatorial method for identifying the known set of hyperplanes.

TS06-2: Day 5, 15:00–15:30, KMB 213 (Chair: Takafumi Mase)

TS06-2-1

A two-parameter deformation of the Charlier polynomials: bridging combinatorics and probability

Nobuhiro Asai[1], Marek Bożejko[2], Lahcen Oussi[3], Hiroaki Yoshida[4]

[1]Aichi University of Education, Aichi, Japan
[2]University of Wrocław, Wrocław, Poland
[3]Wrocław University of Science and Technology, Wrocław, Poland
[4]Josai University and Ochanomizu University, Tokyo, Japan

Email: nasai [at] auecc.aichi-edu.ac.jp, hyoshida [at] josai.ac.jp

We study a two-parameter deformation of the one-variable Charlier polynomials, called the (q, t)-Charlier polynomials, obtained from a deformation of the classical Poisson law via methods of noncommutative probability on a deformed free Fock space. These polynomials extend the q-Charlier polynomials of Saitoh-Yoshida (2000) by introducing an additional parameter t, leading to richer algebraic and combinatorial structures. We show that noncommutative probabilistic methods provide a useful framework for constructing and analyzing such orthogonal polynomials and their combinatorial models. In particular, we derive explicit moment formulas for the associated Poisson-type deformation via a card arrangement model encoding set partitions together with crossing and nesting statistics. The resulting expressions naturally exhibit a duality between crossings and nestings, governed by structures related to generalized Fibonacci numbers. Our results provide a bridge between combinatorial and noncommutative probabilistic interpretations, while yielding a two-parameter extension of the classical Charlier polynomials. This is based on the paper arXiv:2508.12659.

TS07: Quantum Walks and Quantum Information

Coordinator: Ada Chan

TS07-1: Day 2, 10:30–12:00, KMB 213 (Chair: Sho Kubota)

TS07-1-1

Interpolating walk between quantum and random walks on finite graphs

Etsuo Segawa

Yokohama National University, Yokohama, Japan

Email: segawa-etsuo-tb@ynu.ac.jp

We consider an interpolating walk with a parameter $q \in [0, 1]$ connecting a discrete-time quantum walk ($q = 1$) and its intrinsic persistent random walk ($q = 0$). The time evolution of this interpolating walk is given by the convex combination of CPTP maps for the quantum walk and the persistent random walk. The time evolution can be interpreted as the following dynamics of 2 walkers "correlated" with each other on the same graph: 2 walkers move independently of each other with probability q, while 2 walkers move in the same "direction" with probability $1 - q$ at each time step. The notion of direction in the underlying graph is extended by defining a partition of the symmetric arc set A of the graph and we replace the persistent random walk with its extended walk called the correlated walk. For example, the correlated walk induced by the trivial partition $\pi_0 : A = \{A\}$ coincides with the quantum walk, while the correlated walk induced by the maximal partition $\pi_{\max} : A = \sqcup_{a\in A}\{a\}$ is equivalent to the persistent random walk on the same graph. Moreover, if the underlying graph is arc colorable, then the partition π_c induced by the coloring gives an open quantum random walk. The time evolution operator with partition π and parameter q is not ensured normality but even the diagonalizability in general. However, we show that the spectral radius and the operator norm of the interpolating walk are both equal to 1. Moreover, every eigenvalue on the unit circle in the complex plane is semi-simple and its eigenspace coincides with that of the correlated walk. Using these spectral properties, we discuss the behavior of the interpolating walk in the long time limit.

TS07-1-2

Enabling subspace state transfer with quantum coins

Hanmeng Zhan, Yichi Xu

Worcester Polytechnic Institute, Worcester, USA

Email: hzhan@wpi.edu

A discrete quantum walk is a quantization of the discrete random walk on a graph. In many models, one step of the walk consists of a quantum coin flip followed by a conditional shift. The choice of coin can significantly affect the dynamics of the walk, and can be used to enable transport phenomena that do not occur in a continuous quantum walk. In this talk, I will show how some quantum coins help us send multiple orthogonal states from one vertex to another, enabling transfer of a subspace of states localized at a vertex.

TS07-1-3

Behavior of quantum walk with many weak bridges

Taisuke Hosaka, Etsuo Segawa

Yokohama National University, Yokohama, Japan

Email: hosaka-taisuke-pn@ynu.jp

In this presentation, we will show the dynamics of discrete-time quantum walks on graphs with a structure called weak bridge, a phenomenon referred to as pulsation. For the connected graph G, let B be the edge cut set on G. We put a weight parametrized by $\epsilon > 0$ on the edges belonging to B, while all edges except a bridge are put weight 1. The parameter ϵ is considered as the strength of connectivity. If $\epsilon = 1$, standard Grover walk on the graphs is reproduced. While if $\epsilon = 0$, effectively the bridge disappears, and the graph is decomposed into the connected components of $G - B$. This means that the smaller the value of ϵ, the weaker the connectivity among some connected components. Then, for sufficiently small ϵ, we call the element of B *weak bridges*.

We revealed the finding probabilities on the connected components of $G - B$ are governed by the Chebyshev polynomials of the first kind evaluated at $I - \epsilon L_{sym}$, where L_{sym} is the weighted Laplacian of the graph whose vertices are replaced with connected components of $G \setminus B$. In other words, this states that the number and locations of the bridges have no asymptotic effect on the behavior of the quantum walker; rather, it is determined solely by this structure of the reduced graph and the size of the connected components.

Furthermore, we show the underlying continuous-time unitary dynamics on the total arc space. We first establish that the discrete-time quantum walk is asymptotically approximated by a continuous-time state under the time scale $\tau = t\sqrt{2\epsilon}$. By employing the lifting operators to elevate the vertex state of the induced graph $G - B$, we demonstrate that this induced continuous-time state Ψ_τ is governed by a wave equation driven by the operator $\mathcal{L}$. This operator lifts the spatial structure encoded in L_{sym} to the total arc space.

TS07-2: Day 2, 15:00–16:30, KMB 213 (Chair: Hanmeng Zhan)

TS07-2-1

Pretty good fractional revival on abelian Cayley graphs

Akash Kalita, Bikash Bhattacharjya

Indian Institute of Technology Guwahati, Guwahati, India

Email: akash.kalita@iitg.ac.in, b.bikash@iitg.ac.in

Let Γ be a graph with the adjacency matrix A. The transition matrix of Γ is denoted by $H(t)$ and is defined as $H(t) := \exp(-\mathbf{i}tA)$, where $\mathbf{i} := \sqrt{-1}$ and t is a real variable. The graph Γ is said to exhibit fractional revival (FR in short) between the vertices a and b if there exists a positive real number t such that $H(t)\mathbf{e}_a = \alpha\mathbf{e}_a + \beta\mathbf{e}_b$, where $\alpha, \beta \in \mathbb{C}$ such that $\beta \neq 0$ and $|\alpha|^2 + |\beta|^2 = 1$. The graph Γ is said to exhibit pretty good fractional revival (PGFR in short) between the vertices a and b if there exists a sequence of real numbers $\{t_k\}$ with $\lim_{k\to\infty} H(t_k)\mathbf{e}_a = \alpha\mathbf{e}_a + \beta\mathbf{e}_b$, where $\alpha, \beta \in \mathbb{C}$ such that $\beta \neq 0$ and $|\alpha|^2 + |\beta|^2 = 1$. In the definition of PGFR, if $\alpha = 0$ then Γ is said to exhibit pretty good state transfer (PGST in short) between a and b. In this paper, we obtain some sufficient conditions for circulant graphs exhibiting PGFR. We also find some sufficient conditions for non-circulant abelian Cayley graphs exhibiting PGFR. From these sufficient conditions, we find infinite families of circulant graphs and non-circulant abelian Cayley graphs exhibiting PGFR that fail to exhibit FR and PGST. Finally, we obtain some necessary conditions for some families of circulant graphs exhibiting PGFR. Some of our results generalize the results of Chan et al. [Pretty good quantum fractional revival in paths and cycles. *Algebr. Comb.* 4(6) (2021), 989-1004.] for cycles.

TS07-2-2

Pretty good state transfer in Grover walks on graphs

Koushik Bhakta, Bikash Bhattacharjya

Indian Institute of Technology Guwahati, Guwahati, India

Email: b.koushik@iitg.ac.in, b.bikash@iitg.ac.in

In this work, we study pretty good state transfer (PGST) in Grover walks on graphs. We consider the transfer of quantum states localized at the vertices of a graph and use Chebyshev polynomials to analyze PGST between such states. In general, we find a necessary and sufficient condition for the occurrence of PGST on graphs. We then focus our analysis on abelian Cayley graphs and derive a necessary and sufficient condition for the occurrence of PGST on such graphs. Consequently, we obtain a complete characterization of PGST on unitary Cayley graphs. Our results yield infinite families of graphs that exhibit PGST yet do not exhibit perfect state transfer.

TS07-2-3

Exact solutions of open quantum Brownian motions on the real line for two-level systems

Manuel Domínguez de la Iglesia[1], Carlos F. Lardizabal[2]

[1]Universidad de Alcalá, Madrid, Spain
[2]Universidade Federal do Rio Grande do Sul, Porto Alegre, Brazil

Email: manuel.dominguezi@uah.es

We study open quantum Brownian motions as quantum counterparts of classical diffusion processes interacting with an external environment. Starting from the microscopic model introduced by I. Sinayskiy and F. Petruccione, we revisit the associated master equation and formulate it as a generalized matrix-valued parabolic system. By combining Fourier transform techniques with spectral methods, we obtain explicit analytical solutions for one-dimensional evolutions of particles possessing a two-level internal degree of freedom.

TS07-3: Day 2, 16:45–17:45, KMB 213 (Chair: Etsuo Segawa)

TS07-3-1

Analytical asymptotics of quantum spatial search on 2D lattices with regular range-m edges

Satoshi Watanabe[1], Yusuke Ide[2], Norio Konno[2], Kei Saito[2]

[1]KDDI Research, Inc., Saitama, Japan
[2]Nihon University, Tokyo, Japan

Email: xsas-watanabe@kddi.com

A quantum walk is the quantum analogue of a random walk with internal (coin) degrees of freedom, and discrete-time coined walks provide a basic framework for spatial search on graphs. On the two-dimensional periodic lattice, coined search exhibits nontrivial behavior and demands careful spectral analysis [1]. Numerical studies further suggest that adding long-range edges can substantially improve 2D search performance, but analytic treatment becomes difficult when such edges break translational symmetry [2].

In this work, we consider a translationally invariant 2D torus augmented with regular range-m edges: each vertex (x, y) is additionally connected to $(x \pm m, y)$ and $(x, y \pm m)$ (modulo the system size). We explicitly construct eigenvectors and eigenphases of the corresponding unperturbed Grover walk with a flip-flop shift, and then introduce a single marked vertex via an SKW-type modification. Using spectral perturbation techniques in the spirit of coined quantum-walk search [1], we derive analytic (asymptotic) estimates/bounds for the search time and success probability as functions of N and m. This provides a tractable benchmark for understanding how controlled long-range structure reshapes the spectrum in 2D search. We are currently investigating the regime where m grows with N.

References

[1] A. Ambainis, J. Kempe, and A. Rivosh, "Coins Make Quantum Walks Faster," arXiv:quant-ph/0402107 (2004).
[2] P. R. Giri, "Quantum walk search on a two-dimensional grid with extra edges," Int. J. Theor. Phys. 62, 121 (2023).

TS07-3-2

Implementation and performance evaluation of CVaR-QAOA for ternary portfolio optimization

Shintaro Yamamura[1], Satoshi Watanabe[2] , Masaya Kunimi[1], Kazuhiro Saito[2], Tetsuro Nikuni[1]

[1]Tokyo University of Science, Tokyo, Japan
[2]KDDI Research, Inc., Saitama, Japan

Email: 1225570@ed.tus.ac.jp

The Quantum Approximate Optimization Algorithm (QAOA) [1] is a promising approach for solving combinatorial optimization problems on gate-based quantum computers. In standard QAOA, the variational parameters are typically optimized to minimize the expectation value of the cost Hamiltonian. Recent studies have shown that Conditional Value-at-Risk-based QAOA, or CVaR-QAOA, which evaluates the average cost over the lowest α-fraction of measurement outcomes, can improve convergence and solution quality in binary optimization models [2]. However, the optimization behavior and performance of CVaR-QAOA in ternary optimization problems have not yet been fully investigated.

In this study, we apply CVaR-QAOA to ternary portfolio optimization with "long," "neutral," and "short" positions [3] and compare its performance with expectation-value-based optimization. Using numerical simulations, we systematically evaluate the robustness of the optimization and its dependence on the CVaR parameter α, demonstrating the effectiveness of CVaR-QAOA for ternary optimization problems and providing insights into its behavior in high-dimensional discrete search spaces.

References

[1] E. Farhi, J. Goldstone, and S. Gutmann, arXiv:1411.4028 (2014).
[2] P. K. Barkoutsos, et al., Quantum 4, 256 (2020).
[3] S. Yamamura, et al., arXiv:2602.21562 (2026).

TS08: Asymptotics of Orthogonal Polynomials and Special Functions

Coordinator: Andrei Martínez-Finkelshtein

TS08-1: Day 4, 15:00–16:30, KMB 211 (Chair: Andrei Martínez-Finkelshtein)

TS08-1-1

A Riemann-Hilbert problem for Jacobi-Pineiro orthogonal polynomials

Sergei Kalmykov[1], Vladimir Lysov[2], Vinay Shukla[1]

[1]Shanghai Jiao Tong University, Shanghai, China
[2]Keldysh Institute of Applied Mathematics, Moscow, Russia

Email: vinayshukla4321@gmail.com

We investigate the asymptotic behaviour of Jacobi-Pineiro polynomials of degree $2n$ orthogonal on $[0, 1]$ with respect to weights $w_j(x) = x^{\alpha_j}(1-x)^{\beta}$, $j = 1, 2$ where $\alpha_1, \alpha_2, \beta > -1$, and $\alpha_1 - \alpha_2 \in (0, 1)$. These polynomials are characterized by a Riemann-Hilbert problem for a 3×3 matrix-valued function. We use the Deift-Zhou steepest descent method for Riemann-Hilbert problems to obtain strong uniform asymptotics in the complex plane. The local parametrix around the origin is constructed using Meijer G-functions. We match the local parametrix around the origin with the global parametrix with a double matching, a technique that was recently introduced.

TS08-1-2

Bipolar Green function and the asymptotics of Cauchy biorthogonal polynomials

Vladimir Lysov

Keldysh Institute of Applied Mathematics, Moscow, Russia

Email: vlysov@mail.ru

We discuss the asymptotic behavior of Cauchy biorthogonal polynomials. The corresponding orthogonality weights may depend on the degree of the polynomials. Then, the limit distribution of the zeros of these polynomials is described by a vector measure that minimizes the energy functional in the external field.

Furthermore, the support of the extremal measure also minimizes the value of a certain functional on compact sets — a vector analog of the Mhaskar–Saff functional. This extremal problem is formulated in terms of vector analogs of the logarithmic capacity and Robin measure. The main result is that both of these quantities can be calculated using the bipolar Green's function on the corresponding Riemann surface. We will demonstrate this result using the example of Hermitian weights.

The talk is based on joint works with Alexander Dyachenko, Sergey Kalmykov and Maria Lapik.

TS08-1-3

On a Szegő theorem for extremal polynomials

Gökalp Alpan[1], Maxim Zinchenko[2]

[1]Sabancı University, Istanbul, Turkey
[2]University of New Mexico, Albuquerque, USA

Email: gokalp.alpan@sabanciuniv.edu, maxim@math.unm.edu

A useful starting point for Szegő-type questions on the real line is the finite-gap setting. Christiansen, Simon, and Zinchenko [3] considered the Szegő condition on the absolutely continuous part of the measure, a Blaschke-type condition on mass points outside the support, and boundedness of the corresponding normalized Widom factors. Christiansen [2] extended this circle of ideas to Parreau–Widom sets, bringing the Parreau–Widom property of the complement into the discussion as another natural feature of the problem. Taken together, these works provide a natural framework for the asymptotic study of extremal polynomials on compact subsets of $\mathbb{R}$.

In this talk we discuss how the perspective of [2, 3] can be carried over to extremal polynomials and describe a Szegő-type picture for real compact sets. The talk will focus on ongoing joint work [1] on the role of the above conditions in the asymptotic theory of extremal polynomials.

References

[1] G. Alpan and M. Zinchenko, *Szegő's theorem for extremal polynomials on subsets of* $\mathbb{R}$, in preparation.

[2] J. S. Christiansen, *Szegő's theorem on Parreau–Widom sets*, Adv. Math. **229** (2012), 1180–1204.

[3] J. S. Christiansen, B. Simon, and M. Zinchenko, *Finite gap Jacobi matrices, II. The Szegő class*, Constr. Approx. **33** (2011), 365–403.

TS08-2: Day 4, 16:45–18:15, KMB 211 (Chair: Vladimir Lysov)

TS08-2-1

Chebyshev polynomials related to Jacobi weights

Jacob S. Christiansen[1], Olof Rubin[2]

[1]Lund University, Sweden
[2]Royal Institute of Technology, Stockholm, Sweden

Email: jacob_stordal.christiansen@math.lth.se

Among all monic polynomials of degree n, the classical Jacobi polynomials minimize the L^2-norm with respect to the weight $(1-x)^\alpha(1+x)^\beta$ on $[-1, 1]$. But what happens if we minimize the weighted sup-norm instead? Do these two classes of polynomials share common features, such as their asymptotics?

In this talk, I will explore these sup-norm minimizers—which we refer to as Chebyshev Jacobi polynomials—focusing on the monotonicity of their Widom factors $\mathcal{W}_n(\rho_\alpha, \rho_\beta)$, where $\rho_\alpha = \alpha/2 + 1/4$ and $\rho_\beta = \beta/2 + 1/4$. Building on classical work by S. N. Bernstein, we rigorously settle the asymptotic and monotonic behavior for most of the parameter space. Our approach leverages a Bernstein-type inequality alongside an elegant technique connecting Chebyshev Jacobi polynomials to weighted Chebyshev polynomials on the unit circle.

This analysis, however, reveals a fascinating open problem for ''mixed'' parameter regimes (e.g., $0 < \rho_\alpha < 1/2 \le \rho_\beta$). In this region, the exact monotonic behavior remains theoretically unresolved. To address this, I will present striking numerical evidence—obtained via a generalized complex Remez algorithm—revealing a phase transition from monotonically increasing to decreasing. Guided by theoretical asymptotic bounds, this shift occurs in the close vicinity of the circular arc $(\rho_\alpha - 1/4)^2 + (\rho_\beta - 1/4)^2 = 1/8$, though our numerics suggest the true boundary is a distinct, unknown curve. Rigorously proving the existence and exact location of this phase transition remains a compelling open challenge.

References

[1] J. S. Christiansen and O. Rubin, *Chebyshev Polynomials Related to Jacobi Weights*, Int. Math. Res. Not. IMRN, **2025**(18), 1–23.

TS08-2-2

Chebyshev and Faber polynomials on curves with corners and cusps

Erwin Miña-Díaz[1], Olof Rubin[2], Aron Wennman[3]

[1]The University of Mississippi, Oxford, USA
[2]KTH Royal Institute of Technology, Stockholm, Sweden
[3]Katholieke Universiteit Leuven, Leuven, Belgium

Email: minadiaz@olemiss.edu

The Faber polynomials were introduced in 1903 in order to provide an explicit series representation for analytic functions on a Jordan domain. In 1920 they were shown to have asymptotically minimal supremum norm on the domain among all polynomials with the same leading coefficient. The actual minimizers are the Chebyshev polynomials, and Faber's observation shows that the two sequences become asymptotically comparable. This minimality result was later proved for domains with sufficiently smooth boundaries, but the presence of corners breaks the classical arguments.

We will present a new construction of weighted Faber polynomials that recovers asymptotic minimality for domains with corners and even cusps. This allows us to determine the asymptotic behavior of the associated Chebyshev polynomials. We will also describe the uniform behavior of Faber polynomials in neighborhoods of corner singularities.

TS08-2-3

Chebyshev polynomials on a Jordan arc

Benedikt Buchecker[1], Benjamin Eichinger[2], Olof Rubin[3], Aron Wennman[1]

[1]KU Leuven, Belgium
[2]Lancaster University, United Kingdom
[3]KTH Stockholm, Sweden

Email: benedikt.buchecker@kuleuven.be

Given a compact set K in the plane, the Chebyshev polynomials for K are defined as the monic polynomials with minimal supremum norm over K. Understanding the connection between the geometry of K and analytic properties of the Chebyshev polynomials is a classical topic. The fundamental case when K is an analytic (closed) Jordan curve was understood by Faber more than a century ago. Nearly 50 years later, Widom substantially extended the theory by considering K to be a finite union of disjoint smooth Jordan curves.

This talk is about the case when K is a Jordan arc, which has surprisingly proven more elusive. Using weighted Faber polynomials and extremal signatures, we are able to show asymptotics of the norms ("Widom factors") of Chebyshev polynomials on an analytic Jordan arc. In this case, we give an affirmative answer to a conjecture posed by Christiansen, Simon, and Zinchenko, refining the predictions made by Widom in 1969.

This reports on ongoing work with Benjamin Eichinger, Olof Rubin and Aron Wennman.

TS08-3: Day 5, 10:30–11:30, KMB 211 (Chair: Hideshi Yamane)

TS08-3-1

Infinitely divisible modified Bessel distributions

Árpád Baricz[1,2]

[1]Babeş-Bolyai University, Cluj-Napoca, Romania
[2]Óbuda University, Budapest, Hungary

Email: bariczocsi@yahoo.com

In many applications, random effects can be modeled as sums of independent components with the same distribution, naturally leading to the concept of infinite divisibility. Such models arise in areas including biology, physics, economics, and insurance. This talk focuses on continuous probability distributions on $[0, \infty)$, including McKay, K-, generalized inverse Gaussian, and generalized McKay distributions, which are closely related to modified Bessel functions of the first and second kinds. We show that many of these distributions belong to important classes such as infinitely divisible and self-decomposable distributions, generalized gamma convolutions, and distributions with hyperbolically completely monotone densities. Moreover, by using the representation and inversion theorems for Stieltjes transforms as well as some integral representations and asymptotics for Gaussian and Tricomi hypergeometric functions, modified Bessel functions, respectively, we derive new Stieltjes transform representations and obtain new infinitely divisible modified Bessel distributions. We also present a new proof of the infinite divisibility of the ratio of two gamma random variables based on the Pick function characterization theorem.

The talk is based on the following paper: Á. Baricz, D.K. Prabhu, S. Singh, V.A. Vijesh, Infinitely divisible modified Bessel distributions, *Pacific Journal of Mathematics* (in press), `https://arxiv.org/abs/2406.17721`.

TS08-3-2

On vector and branching continued fractions

Alexander Dyachenko, Vladimir Lysov

Keldysh Institute of Applied Mathematics (RAS), Moscow, Russia

Email: diachenko@sfedu.ru

Being a critical tool for rational approximations and moment problems, Stieltjes and Jacobi continued fraction proved to be important in many fields. A careful analysis of combinatorial aspects of such fractions was conducted by Flajolet in 1980. Later on, continued fractions of some other types (e.g. those of Thron type arising in two-point rational approximations) were interpreted in a similar way.

Pétréolle, Sokal and Zhu recently found an impressive application of branching continued fractions of Stieltjes, Jacobi and Thron types: besides combinatorial interpretations of the related objects and questions of total positivity, they gave several families of nice formulae for ratios of generalised hypergeometric series.

At the same time, in various works on asymptotic analysis, one meets the so-called vector continued fractions. They originate from works of Jacobi, and have been studied in detail by Perron. The aim of this talk is to trace in detail the relationship between branched and vector continued fractions of Stieltjes, Jacobi and Thron types.

TS08-4: Day 5 15:00–16:00, KMB 211 (Chair: Árpád Baricz)

TS08-4-1

Asymptotic expansions of finite Hankel transforms and the surjectivity of convolution operators

Yasunori Okada[1], Hideshi Yamane[2]

[1]Chiba University, Chiba, Japan
[2]Kwansei Gakuin University, Hyogo, Japan

Email: yamane@kwansei.ac.jp

A compactly supported distribution is called invertible in the sense of Leon Ehrenpreis and Lars Hörmander if convolution with it induces a surjective map on $C^\infty(\mathbb{R}^n)$. Well-known examples include distributions of the form $P(D)\delta(x-a)$, where $P(D)$ is a linear partial differential operator with constant coefficients and $a \in \mathbb{R}^n$ is a fixed vector. We give sufficient conditions for radial functions to be invertible. Our analysis is based on asymptotic expansions of finite Hankel transforms. The leading term may arise either from the origin or from the boundary of the support of the function. By combining a formula due to Roderick Wong with one of our own derived from a classical formula by Nikolay Yakovlevich Sonin, we obtain asymptotic expansions of finite Hankel transforms of functions having singularities at points other than the origin.

TS08-4-2

Asymptotics of weighted Bergman polynomials

Lennart A. Huebner

KU Leuven, Leuven, Belgium

Email: lennart.huebner@kuleuven.be

We study strong asymptotics of planar orthogonal polynomials with respect to fixed weighted area measures on Jordan domains. The aim is to understand how the geometry of the domain and the regularity of the weight enter the large-degree behavior of these polynomials.

The result interpolates between two earlier results: a classical theorem of Suetin which gives the leading order asymptotics for Hölder-continuous weights, and a more recent theorem of Hedenmalm–Wennman which gives a full asymptotic expansion for analytic weights. For weights with intermediate degrees of smoothness, only Suetin's leading order expansion was available. We show that, under finite smoothness assumptions on both the domain and the weight, the orthogonal polynomials admit an asymptotic expansion of finite length, valid in the exterior region and in a shrinking neighborhood of the boundary.

TS08-4-3

Orthogonality relations for the symmetric polynomials in the q^{-1}-Askey scheme

Howard S. Cohl[1], Roberto S. Costas-Santos[2], Xiang-Sheng Wang[3]

[1]NIST, Gaithersburg, USA
[2]Universidad Loyola Andalucía, Seville, Spain
[3]University of Louisiana at Lafayette, Lafayette, USA

Email: howard.cohl@nist.gov, rscosa@gmail.com, xswang@louisiana.edu

In this talk we summarize known infinite families of orthogonality relations for the symmetric, q^{-1}-symmetric and dual subfamilies of the Askey–Wilson polynomials. These polynomials are the continuous dual q and q^{-1}-Hahn polynomials, the q and q^{-1}-Al-Salam–Chihara polynomials, the continuous big q and q^{-1}-Hermite polynomials and the continuous q and q^{-1}-Hermite polynomials and their dual counterparts which are the big q-Jacobi polynomials, little q-Jacobi polynomials and the q and q^{-1}-Bessel polynomials. The q^{-1}-symmetric polynomials in the q-Askey scheme (except the continuous big q^{-1}-Hermite polynomials) satisfy an indeterminate moment problem, implying an infinite number of orthogonality relations with positive measure. Among the infinite number of orthogonality relations for the q^{-1}-symmetric families, we attempt to summarize those currently known. These fall into several classes, including continuous orthogonality relations and infinite discrete (including bilateral) orthogonality relations. Using symmetric limits, we derive a new infinite discrete orthogonality relation for the continuous big q^{-1}-Hermite polynomials with non-positive weight functions. Using duality relations, we explore orthogonality relations for and from the dual families associated with the q and q^{-1}-symmetric subfamilies of the Askey–Wilson polynomials. In order to give a complete description of the convergence properties for these polynomials, we provide large degree asymptotics using Darboux's method. In order to apply Darboux's method, we derive a generating function with two free parameters for the q^{-1}-Al-Salam–Chihara polynomials.

TS09: Multiple Orthogonal Polynomials

Coordinator: Walter Van Assche

TS09-1: Day 2 10:30–12:00, KMB 211 (Chair: Walter Van Assche)

TS09-1-1

Hahn-classical $(r+1)$-fold symmetric r-orthogonal polynomials

Hélder Lima

Universidade de Aveiro, Portugal

Email: helder.lima@ua.pt

Let r be a positive integer. A polynomial sequence $(P_n(x))_{n\in\mathbb{N}}$ is r-orthogonal if it is formed by the type II multiple orthogonal polynomials for the multi-indices on the step-line with respect to a system of r measures (or linear functionals) and is $(r+1)$-fold symmetric if $P_n\left(\mathrm{e}^{\frac{2\pi i}{r+1}}x\right) = \mathrm{e}^{\frac{2\pi i n}{r+1}}P_n(x)$ for all $n\in\mathbb{N}$. A polynomial sequence $(P_n(x))_{n\in\mathbb{N}}$ is a $(r+1)$-fold symmetric r-orthogonal polynomial sequence if and only if it satisfies a three-term recurrence relation of order $r+1$ of the form

$$P_{n+r+1}(x) = x\,P_{n+r}(x) - \alpha_n\,P_n(x), \quad \alpha_n \neq 0, \quad \text{for all } n\in\mathbb{N},$$

with initial conditions $P_j(x) = x^j$ for $0\leq j\leq r$. A r-orthogonal polynomial sequence $(P_n(x))_{n\in\mathbb{N}}$ is said to be Hahn-classical if its sequence of derivatives $\left(\frac{1}{n+1}P'_{n+1}(x)\right)_{n\in\mathbb{N}}$ is also r-orthogonal.

We obtain a thorough characterisation of all Hahn-classical $(r+1)$-fold symmetric r-orthogonal polynomial sequences for general r: we present explicit representations for the polynomials as terminating hypergeometric series, we study their differential properties, formulas for their recurrence coefficients, and the location of their zeros, and we describe their orthogonality measures, which are supported on a star-like set on the complex plane where the zeros of the polynomials are located.

When $r=1$, these polynomials reduce to the classical and symmetric Hermite and Gegenbauer orthogonal polynomials; when $r=2$ the Hahn-classical 3-fold symmetric 2-orthogonal polynomials were firstly studied by Douak and Maroni and later characterised in detail by Loureiro and Van Assche.

TS09-1-2

On families of discrete multiple orthogonal polynomials on a star-like set

Jorge Arvesú[1], Alejandro J. Quintero-Roba[2]

[1]University Carlos III of Madrid, Madrid, Spain
[2]Baylor University, Texas, USA

Email: jarvesu@math.uc3m.es

We discuss several families of Angelesco multiple orthogonal polynomials defined by shared orthogonality conditions with respect to a system of weight functions supported on a star-like set. The focus is on structural properties, including raising operators, Rodrigues-type formulas, and nearest-neighbor recurrence relations. We also address several limit relations.

References

[1] J. Arvesú, J. Coussement, and W. Van Assche, "Some discrete multiple orthogonal polynomials," J. Comput. Appl. Math. **153**, 19–45 (2003).

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[3] M. Leurs and W. Van Assche, "Laguerre–Angelesco multiple orthogonal polynomials on an r-star," J. Approx. Theory **250**, 105324, 1–30 (2020).

TS09-1-3

Jacobi-Piñeiro multiple orthogonal polynomials on the simplex

Juan Antonio Villegas[1], Lidia Fernández[1], Ana Foulquié-Moreno[2]

[1]Universidad de Granada. Granada, Spain
[2]Universidade de Aveiro. Aveiro, Portugal

Email: jantoniovr@ugr.es, lidiafr@ugr.es, foulquie@ua.pt

One of the most classical families of multiple orthogonal polynomials is the Jacobi-Piñeiro family, which extends the Jacobi polynomials and their properties to the multiple orthogonal setting. In particular, they can be computed in terms of a Rodrigues-type formula. This formula can be expressed as a concatenation of differential operators. In this talk, we explain how this property allows us to extend the Jacobi-Piñeiro family to the multivariate simplex, and to obtain a Rodrigues-type formula for this extension. Finally, we will show some applications of these polynomials to approximation of functions.

TS09-2: Day 2 15:00–16:30, KMB 211 (Chair: Walter Van Assche)

TS09-2-1

Gaussian quadrature rules for multiple orthogonal polynomials

Teresa Laudadio[1], Nicola Mastronardi[1], Walter Van Assche[2], Paul Van Dooren[3]

[1]Istituto per le Applicazioni del Calcolo, CNR, Bari, Italy
[2]KU Leuven, Belgium
[3]Catholic University of Louvain, Belgium

Email: teresa.laudadio@cnr.it

We focus on the computation of simultaneous Gaussian quadrature rules associated to multiple orthogonal polynomials (MOP). Given r weight functions $w^{(i)}(x) \geq 0$, with support Δ_i, $i = 1, \ldots, r$, on the real line, the MOP $\{p_n(x)\}_{n=0}^{\infty}$ satisfy the following orthogonality conditions:

$$\int_{\Delta_i} p_n(x)x^k w^{(i)}(x)dx = 0, \quad 0 \leq k \leq n_{i-1}, \quad \text{with} \quad n = \sum_{i=1}^{r} n_i.$$

The nodes x_j and weights $\omega_j^{(i)}$ of a simultaneous Gaussian quadrature rule $\sum_{j=1}^{n} f(x_j)\omega_j^{(i)} \approx \int_{\Delta_i} f(x)w^{(i)}(x)dx$, $1 \leq i \leq r$, can be computed via the eigendecomposition of a banded lower Hessenberg matrix H_n, whose entries are the coefficients of recurrence relations associated to the corresponding MOP. Nevertheless, the eigenvalues of H_n are often very ill–conditioned, and stable methods such as variants of the Golub–Welsch algorithm, yield unreliable results.

Here, we propose a new balancing procedure that drastically reduces the condition of the Hessenberg eigenvalue problem, allowing to compute the simultaneous Gaussian quadrature rule in a reliable way for different kinds of MOP, requiring $O(n^2)$ computational complexity and $O(n)$ memory, in floating point arithmetic.

TS09-2-2

Recurrence updates and downdates for multiple orthogonal polynomials

Amin Faghih, Marc Van Barel, Raf Vandebril

KU Leuven, Leuven, Belgium

Email: amin.faghih@kuleuven.be, marc.vanbarel@kuleuven.be, raf.vandebril@kuleuven.be

We study multiple orthogonal polynomials associated with several positive discrete measures on the real line. Focusing on type I and type II multiple orthogonal polynomials and their step-line recurrence relations, we address the inverse eigenvalue problem of reconstructing the corresponding banded upper Hessenberg recurrence matrix from given nodes and weights.

We propose efficient updating and downdating procedures that modify an existing recurrence matrix when nodes are added to or removed from the underlying discrete measures, avoiding the need to recompute the full polynomial basis. The updating strategy is based on structured nonsingular eliminations, while the downdating procedure relies on an eigenvalue deflation technique.

TS09-2-3

Zeros of multiple orthogonal polynomials

Rostyslav Kozhan, Marcus Vaktnäs

Uppsala University, Sweden

Email: kozhan@math.uu.se

We demonstrate how the Christoffel transform serves as a tool to analyze the zeros of type I and type II multiple orthogonal polynomials. We apply this method to Nikishin systems on the real line, producing location and interlacing results for zeros of type I polynomials. Furthermore, we extend this methodology to the unit circle, where we analyze both multiple orthogonal and paraorthogonal polynomials for Angelesco and AT systems.

TS09-3: Day 2 16:45–18:15, KMB 211 (Chair: Walter Van Assche)

TS09-3-1

Szegő mapping and Geronimus relations for multiple orthogonal polynomials

Rostyslav Kozhan, Marcus Vaktnäs

Uppsala University, Sweden

Email: marcus.vaktnas@math.uu.se, marcusva96@gmail.com

We investigate Laurent polynomials that satisfy simultaneous orthogonality relations with respect to several measures supported on the unit circle. This construction is alternative to that of Minguez and Van Assche from [2]. For our construction, previous work identified analogue results to the perfectness and zero location of Angelesco and AT systems on the real line.

In this talk we generalize the Szegő mapping relations to multiple orthogonal polynomials. We also find generalized Szegő recurrence relations for the associated Laurent polynomials, which leads to Geronimus relations for the nearest-neighbour coefficients of multiple orthogonal polynomials on the real line. Through these relations we can compute the multiple orthogonal orthogonal polynomials on the real line and their nearest-neighbour recurrence coefficients, as well as establish perfectness on the real line. The talk is based on the paper [1], joint work with Rostyslav Kozhan.

References

[1] R. Kozhan, M. Vaktnäs, *Szegő mapping and Hermite–Padé polynomials for multiple orthogonality on the unit circle*, arXiv:2601.04783.

[2] J. Mínguez Ceniceros, W. Van Assche, *Multiple orthogonal polynomials on the unit circle*, Constr. Approx., 28 (2008), 173–197.

TS09-3-2

Asymptotics of discrete multiple orthogonal polynomials

Tomas Lasic Latimer

The University of Sydney, New South Wales, Australia

Email: tomas.lasiclatimer@sydney.edu.au

This talk focuses on recent results concerning the asymptotics of multiple q-orthogonal polynomials as their degree tends to infinity. In particular, we consider the case where there are two orthogonality weights supported on the q-lattice (q^k for $k \in \mathbb{N}$). We detail interesting challenges and observations that arise in the 3×3 Riemann Hilbert Problem we are confronted with in the analysis of these multiple q-orthogonal polynomials. These challenges include a loss in linear independence of the rows and a different estimate for the order of the error compared to classical q-orthogonal polynomials.

TS09-3-3

Strong asymptotics of weighted orthogonal polynomials

Alexander Aptekarev[1], Sergey Denisov[2], Maxim Yattselev[3]

[1]Keldysh Institute for Applied Mathematics, Moscow, Russian Federation
[2]University of Wisconsin, Madison, Wisconsin, USA
[3]Indiana University Indianapolis, Indianapolis, Indiana, USA

Email: maxyatts@iu.edu

Let $\vec{\mu} = (\mu_1, \ldots, \mu_d)$ be a vector of d compactly supported Borel measures on the real line. Denote by $[\alpha_i, \beta_i]$ the convex hull of the support of μ_i. Assume that these measures form an Angelesco system, i.e., the intervals $[\alpha_i, \beta_i]$ are pairwise disjoint. Given a multi-index $\vec{n} = (n_1, \ldots, n_d)$, where n_i are non-negative integers, type II multiple orthogonal polynomials w.r.t. $\vec{\mu}$ corresponding to $\vec{n}$, is a monic polynomial $P_{\vec{n}}(x)$ of degree $|\vec{n}| = n_1 + \cdots + n_d$ satisfying

$$\int x^k P_{\vec{n}}(x)\, d\mu_i(x) = 0, \quad k \in \{0, \ldots, n_i - 1\}, \quad i \in \{1, \ldots, d\}.$$

It was shown by Angelesco that $P_{\vec{n}}(x) = P_{\vec{n},n_1}(x) \cdots P_{\vec{n},n_d}(x)$, where each $P_{\vec{n},n_i}(x)$ has exactly n_i zeros on $[\alpha_i, \beta_i]$. Clearly,

$$\int x^k P_{\vec{n},n_i}(x) w_{\vec{n},n_i}(x)\, d\mu_i(x) = 0, \quad k \in \{0, \ldots, n_i - 1\}, \quad w_{\vec{n},n_i}(x) = \prod_{j \neq i} |P_{\vec{n},n_j}(x)|.$$

Hence, $P_{\vec{n},n_i}(x)$ can be viewed as a weighted orthogonal polynomial. In this talk I will discuss results on the strong asymptotics of weighted orthogonal polynomials needed to prove strong asymptotics of $P_{\vec{n}}(x)$ as $|\vec{n}| \to \infty$.

TS09-4: Day 3 10:30–12:00, KMB 211 (Chair: Maxim Yattselev)

TS09-4-1

Non-commutative multiple bi-orthogonal polynomials: formal approach and integrability

Adam Doliwa

University of Warmia and Mazury, Olsztyn, Poland

Email: doliwa@matman.uwm.edu.pl

We define the non-commutative multiple bi-orthogonal polynomial systems, which simultaneously generalize the concepts of multiple orthogonality, matrix orthogonal polynomials and of the bi-orthogonality. We present quasideterminantal expressions for such polynomial systems in terms of formal bi-moments. The normalization functions for such monic polynomials satisfy the non-commutative Hirota equations, while the polynomials themselves provide solution of the corresponding linear system. This shows, in particular, that such polynomials form a part of the theory of integrable systems. We study also a specialization of the problem to non-commutative multiple orthogonal polynomials, what results in the corresponding Hankel-type quasideterminantal expressions in terms of the moments. Moreover, such a reduction allows to introduce in a standard way the discrete-time variable and gives rise to an integrable system which is a non-commutative version of the multidimensional discrete-time Toda equations.

TS09-4-2

Multiple orthogonal polynomials of derivative type

Thomas Wolfs

KTH Royal Institute of Technology

Email: wolfs@kth.se

Polynomial ensembles of derivative type play an important role in the study of the eigenvalues of products and sums of random matrices. In this talk, I will discuss several families of multiple orthogonal polynomials that arise as the average polynomials of such ensembles.

First, I will show that many classical families of multiple orthogonal polynomials, including the multiple Hermite, Jacobi-Piñeiro, multiple Charlier and multiple Hahn polynomials, fit naturally into this framework. Each of the mentioned families will correspond to a different kind of derivative type. Afterwards, I will explain how this framework also leads to several new families of multiple orthogonal polynomials. Finally, I will show that such multiple orthogonal polynomials always admit natural (de)composition properties with respect to some polynomial convolution. Such (de)composition properties are particularly useful in the study of zeros of the polynomials. Particular examples of the convolutions that arise in this way include the finite free additive and multiplicative convolution from finite free probability.

TS09-4-3

Multiple orthogonal polynomials with modified Bessel weights

Walter Van Assche

KU Leuven, Belgium

Email: walter.vanassche@kuleuven.be

In 2000 Yakubovich and I introduced multiple orthogonal polynomials (MOPs) with modified Bessel weights $(K_\nu, K_{\nu+1})$. We obtained recurrence relations, differential properties and explicit expressions for these MOPs. A few years later my student Els Coussement and I investigated multiple orthogonal polynomials with modified Bessel weights $(I_\nu, I_{\nu+1})$ and again we obtained recurrence relations, a differential equation and explicit expressions. Both families of MOPs later appeared in applications: the first family is relevant for products of random matrices, the second family turns up in the study of squared Bessel processes. I will explain how the asymptotic behavior of the zeros of these multiple orthogonal polynomials reveals interesting results for these applications.

TS10: Matrix Orthogonal Polynomials

Coordinators: Pablo Román, Erik Koelink

TS10: Day 4 10:30–12:00, KMB 211 (Chair: Walter Van Assche)

TS10-1-1

Shift operators for non-symmetric Heckman–Opdam polynomials and non-symmetric Macdonald–Koornwinder polynomials

Max van Horssen[1], Maarten van Pruijssen[2]

[1]KU Leuven, Leuven, Belgium
[2]Radboud University, Nijmegen, the Netherlands

Email: max.vanhorssen@kuleuven.be

In this talk, we give an algebraic construction of shift operators for the non-symmetric Heckman–Opdam polynomials and the non-symmetric Macdonald–Koornwinder polynomials. To each linear character of the finite Weyl group, we associate forward and backward shift operators, which are differential-reflection and difference-reflection operators that satisfy certain transmutation relations with the (Dunkl-)Cherednik operators. In the Heckman–Opdam case, the construction recovers the non-symmetric shift operators of Opdam and Toledano Laredo for the sign character. As an application, we show that in the Heckman–Opdam case, the non-symmetric shift operators give rise to matrix-valued differential shift operators for a family of multivariate matrix-valued orthogonal polynomials that encode polynomial solutions to the KZ-equations.

TS10-1-2

Matrix valued orthogonal polynomials in random tiling models

Max van Horssen, Arno Kuijlaars

KU Leuven, Leuven, Belgium

Email: max.vanhorssen@kuleuven.be, arno.kuijlaars@kuleuven.be

Matrix valued orthogonal polynomials (MVOP) appear in the analysis of certain random tiling models with periodic weightings. These models have three asymptotic regimes (depending also on boundary conditions) separated by Arctic curves. In the talk it will be explained how the asymptotic analysis of MVOP helps to find the limiting shapes for certain classes of lozenge tilings of a hexagon with weightings that are 3×3 periodic.

TS10-1-3

Matrix-valued Laguerre-type orthogonal polynomials, Darboux transformations and differential operators

Inés Pacharoni

Universidad Nacional de Córdoba, Córdoba, Argentina

Email: ines.pacharoni@unc.edu.ar

The matrix Bochner problem seeks to classify weight matrices whose orthogonal polynomials are eigenfunctions of a second-order differential operator. A natural object in this problem is the algebra $\mathcal{D}(W)$ of all differential operators having the corresponding matrix-valued orthogonal polynomials as eigenfunctions.

In this talk, we study a family of 2×2 Laguerre-type weight matrices $W_{\alpha,\beta}$ which admit a symmetric second-order differential operator D. We analyze the full differential-operator algebra $\mathcal{D}(W_{\alpha,\beta})$.

We prove that $W_{\alpha,\beta}$ is a Darboux transformation of a direct sum of classical scalar Laguerre weights if and only if β is an integer. In this case, the algebra $\mathcal{D}(W_{\alpha,\beta})$ has a rich structure: additional higher-order operators appear beyond D, we give an explicit finite generating set as a $\mathbb{C}[D]$-module and the minimal order at which new generators appear can be arbitrarily large depending on β. When $\beta \notin \mathbb{Z}$, the algebra is simply the polynomial algebra $\mathbb{C}[D]$, yielding explicit irreducible solutions of the matrix Bochner problem which do not arise through Darboux transformations of classical scalar weights.

Joint work with Ignacio Bono Parisi and A. Victoria Torres.

TS11: Orthogonal Polynomials on the Unit Circle

Coordinator: Alexei Zhedanov

TS11: Day 3 10:30–12:00, KMB 212 (Chair: Kazuki Maeda)

TS11-1-1

On the expected number of real level crossings of random polynomials spanned by OPUC

Dustin Newland, Maxim Yattselev

Indiana University Indianapolis, Indianapolis, Indiana

Email: dwnewlan@iu.edu

Consider the degree n polynomial equation $\sum_{i=0}^{n} \eta_i \phi_i(z) = y$, where $\{\phi_i\}_{i=0}^{n}$ are a given set of orthonormal polynomials on the unit circle, $\{\eta_i\}_{i=0}^{n}$ are independent identically distributed random Gaussian variables and $y \in \mathbb{R}$ is fixed. We wish to understand the expected number of real solutions, denoted by $N_n^y(\mathbb{R})$.

Letting $B_0(x) := K_n(x,x)$, $B_1(x) := K_n^{(1,0)}(x,x)$, $B_2(x) := K_n^{(1,1)}(x,x)$, $E_1(x) := \sqrt{B_0(x)B_2(x) - B_1(x)^2}$ be functions defined by the Christoffel-Darboux kernel

$$K_n(z,w) = \sum_{i=0}^{n} \phi_i(z)\phi_i(\bar{w}) = \frac{\phi_n^*(z)\phi_n^*(\bar{w}) - \phi_n(z)\phi_n(\bar{w})}{1 - z\bar{w}}$$

We can write an explicit formula for the expected number of real solutions:

$$\mathbb{E}(N_n^y(\mathbb{R})) = \int_{\mathbb{R}} \frac{yB_1(x)}{\sqrt{2\pi}B_0(x)^{3/2}} \exp\left(-\frac{y^2}{2B_0(x)}\right) \operatorname{erf}\left(\frac{yB_1(x)}{E_1(x)\sqrt{2B_0(x)}}\right) dx + \int_{\mathbb{R}} \frac{E_1(x)}{\pi B_0(x)} \exp\left(-\frac{y^2 B_2(x)}{2E_1(x)^2}\right) dx.$$

What upper and lower bounds can we find for each integral? What do asymptotics look like for large n? How does the class of recurrence coefficient affect these quantities? We will briefly discuss our progress on each of these questions, pointing out the techniques and numerical analysis utilized to reach our results.

TS11-1-2

Modified Camassa–Holm peakons, extreme degenerations into Toda-type lattices, and interpretations in terms of orthogonality

Xiang-Ke Chang

Academy of Mathematics and Systems Science, Chinese Academy of Sciences, China

Email: changxk@lsec.cc.ac.cn

The modified Camassa–Holm (mCH) equation is an intriguing variant of the CH equation, possessing several distinctive features. It is known that the mCH equation admits the conservative peakon solution characterized by a dynamical system, which we call mCH peakon lattice. In this talk, we show that the mCH peakon lattice can be viewed as a general integrable lattice model, in the sense that, some known lattices including the well-known Toda lattice, the Kac–van Moerbeke lattice, the relativistic Toda lattice, as well as the relativistic Lotka–Volterra lattice and the R_I lattice, can unexpectedly be derived from the mCH peakon lattice as extremely degenerate cases. These reduction results are then interpreted in terms of isospectral deformations related to Frobenius–Stickelberger–Thiele polynomials and R_{II} polynomials, along with their extreme degeneration cases including ordinary orthogonal polynomials (OPs), symmetric OPs, Laurent bi-OPs etc.

TS12: Exceptional Orthogonal Polynomials

Coordinators: David Gómez-Ullate, Robert Milson

TS12-1: Day 4 10:30–12:00, KMB 212 (Chair: Yves Grandati)

TS12-1-1

Bispectrality and the ad-conditions

F. Alberto Grünbaum

University of California, Berkeley

Email: grunbaum@math.berkeley.edu

Bispectrality can be seen as an effort to enlarge the class of special functions such as Hermite, Laguerre, Jacobi, Bessel, Airy, etc. that have grown with and become crucial tools in mathematical physics, both classical and quantum, for a very long time.

One looks for eigenfunctions of a differential operator, such as that of Schroedinger, that satisfy differential or difference equations in the spectral parameter.

The ad-conditions, of Lie algebraic nature, were the first tools used to discover non-classical examples with this property about forty years ago. In the meantime other methods have been found to get more examples.

There is no obvious reason to suspect that this search should have connections with several modern pieces of research such as isospectrality, isomonodromic deformations, group representations, integrable non-linear differential and difference equations, etc. And yet, this is the case.

In this talk we show that properly modified versions of the original nilpotency conditions can be a useful tool to use even in some extensions of the original problem. Building a larger set of examples should help in finding new situations that could have interesting math- physics applications.

TS12-1-2

Equivalences for matrix-valued bispectral polynomials

Ignacio N. Zurrián

Universidad de Sevilla, Spain

Email: ignacio.zurrian@fulbrightmail.org

In this talk, we explore the evolution of equivalence relations for matrix-valued orthogonal polynomials satisfying a differential equation. We begin with the foundational framework of reducibility for matrix weights as established in [J. A. Tirao and I. N. Zurrián, *Reducibility of matrix weights*, Ramanujan J. 45 (2018), no. 2, 349–374]. Building upon these initial considerations, we present recent advancements and a new equivalence relation introduced in [I. Bono Parisi, I. Pacharoni, I. Zurrián. *On a new equivalence relation for matrix-valued orthogonal polynomials*, Integral Transforms and Special Functions, (2025) pp. 1–19].

We will discuss how these relations impact our understanding of the structural properties of these matrix-valued bispectral families and the underlying operators.

TS12-1-3

Binomial prolates, tau functions, and point counting

W. Riley Casper

California State University, Fullerton, USA

Email: wcasper@fullerton.edu

The $(N+1)\times(N+1)$ symmetric Pascal matrix T_N is a generalized discrete time and band-limiting operator for the binomial transform and its eigenvectors are generalized discrete prolate spheroidal wave functions which we call binomial prolates. Their generating functions are prolate in that they are simultaneously eigenfunctions of a third-order differential operator and an integral operator over the line $\{z \in \mathbb{C} : \mathrm{Re}(z) = 1/2\}$. For even, positive integers N, we obtain an explicit formula for the generating function of an eigenvector of the symmetric Pascal matrix with eigenvalue 1. This specific generating function can be viewed as a tau function for a certain Toda-like integrable system hierarchy. When $N = p^n - 1$ for an odd prime p, we relate this generating function to counting points on the Legendre elliptic curve $y^2 = x(x-1)(x-z)$ over the finite field $\mathbb{F}_p$.

TS12-2: Day 4 15:00–16:30, KMB 212 (Chair: Satoru Odake)

TS12-2-1

Spectral diagrams and exceptional polynomials

Robert Milson[1], María Ángeles García-Ferrero[2], David Gómez-Ullate[3]

[1]Dalhousie University, Canada
[2]ICMAT, Madrid, Spain
[3]IE University, Madrid, Spain

Email: rmilson@dal.ca, garciaferrero@icmat.es, dgomezullate@faculty.ie.edu

Exceptional ortogonal polynomials are generalizations of classical OP in as much as they arise as solutions of second-order Sturm-Liouville problems. However, unlike their classical counter-parts the families in questions consist of polynomials that are missing a finite number of degrees.

Most of the literature on the theory of exceptional polynomials and operators on the past 10 years has focused on the polynomial spectrum and eigenfunctions. It is now clear that that a complete, systematic approach needs to consider the larger class of quasi-rational (log-derivative is rational) eigenfunctions. Indeed, we argue that a proper characterization of the quasi-rational eigenfunctions of a given exceptional operator and their asymptotic behaviour essentially specifies the operator uniquely. For this reason, we introduce the the key concepts of a quasi-rational spectrum and a spectral diagram.

I will report on some recent work that allows for the full classification of excpetional Jacobi polynomials, including a novel subclass that allows for an aribtrary number of continuous parameters. The classification methodology reduces to two principles: (i) a combinatorial description of all possible spectral diagrams; and (ii) a procedure for mapping a given spectral diagram into the corresponding exceptional operator.

This initial classification is formal in that the orthogonality of the resulting families is formal because the integrals in questions are divergent and the corresponding inner products are semi-definite. Time permitting, I will also discuss the methodology for tackling this obstacle. This consists of (a) definining a procedure for regularizing the divergent integrals in questions, and then (b) a Theorem to the effect that the subclass of regular families can be characterized and classified in terms of a norm-positivity condition.

TS12-2-2

Using quasi-Darboux transformations to construct exceptional matrix polynomials

Antonio J. Durán

Universidad de Sevilla, Spain

Email: duran@us.es

We introduce a method to construct exceptional matrix polynomials. The method uses what we have called quasi-Darboux transformations.

Roughly speaking, a Darboux transformation consists in factorizing a second order differential operator D in the form $D(y) = B(A(y)) + \lambda y$, where A and B are first order differential operators and $\lambda \in \mathbb{R}$, and construct a new second order differential operator $\tilde{D}$ by reversing the order of the factors: $\tilde{D}(y) = A(B(y)) + \lambda y$ (in this scheme, we can consider scalar or matrix-valued operators and functions). Starting with the second order differential operator associated to the classical families of orthogonal polynomials, Darboux transformation is one of the ways to construct exceptional polynomials in the scalar case. The other main method is to use Krall discrete orthogonal polynomials to construct exceptional discrete polynomials (by dualization) and then taking limit as in the Askey tableau.

We will use the terminology *quasi-Darboux* transformation if in the previous scheme, we factorize D in the form $D(y) = B(A(y)) - y\Psi(t)$, where Ψ is not a scalar parameter but a matrix function of t. In this talk, we show that quasi-Darboux transform is a powerful method to deal with the non-commutativity problems that appear when matrix-valued polynomials are considered. A collection of instructive examples will be shown.

This talk is based on the following joint manuscript with I. Bono Parisi and I. Zurrián.

References

[1] I. Bono Parisi, A.J. Durán and I. Zurrián, *Using quasi-Darboux transformations to construct exceptional matrix polynomials*, arXiv:2510.13487 [math.CA].

TS12-2-3

Rational solutions of dressing chains and higher order Painlevé V equations

Yves Grandati

Université de Lorraine

Email: yves.grandati@univ-lorraine.fr

We present a new approach to determine the rational solutions of the higher order Painlevé V equations associated to even periodic dressing chain systems. We obtain new sets of solutions, giving determinantal representations indexed by specific universal characters.

TS12-3: Day 4 16:45–17:45, KMB 212 (Chair: Robert Milson)

TS12-3-1

Recurrence relations for exceptional Legendre polynomials

David Gómez-Ullate[1], Robert Milson[2], Satoshi Tsujimoto[3]

[1]IE University, Spain
[2]Dalhousie University, Canada
[3]Kyoto University, Japan

Email: david.gomezullate@ie.edu, rmilson@dal.ca, tsujimoto.satoshi.5s@kyoto-u.ac.jp

Exceptional Legendre polynomials obtained through confluent Darboux transformations constitute a family of isospectral deformations of the classical Legendre system depending on a continuous parameter. We investigate the recurrence relations satisfied by these deformed orthogonal polynomials, whose coefficients have continuous parameters, and we study their behaviour in the large-deformation limit.

For the family generated from a seed polynomial P_m, the general theory predicts a recurrence relation obtained by multiplication by a polynomial of degree $2m + 2$. We derive explicit formulas for the first two cases. For $m = 0$ we obtain a five-term recurrence relation, while for $m = 1$ we derive an explicit nine-term recurrence relation. In the undeformed limit these relations reduce to the classical three-term recurrence for Legendre polynomials.

We further analyze the limit in which the deformation parameter tends to infinity. In the case $m = 0$, the non-trivial component of the deformation is shown to coincide with a Jacobi family, which provides a direct explanation of the corresponding five-term recurrence. For $m = 1$, the limiting family is no longer classical. We obtain a Wronskian representation involving Jacobi polynomials and quasi-rational eigenfunctions, thereby identifying the resulting system as an exceptional Jacobi family.

These results reveal a natural connection between Legendre, Jacobi and exceptional Jacobi polynomials and show that continuous isospectral deformations provide a useful framework for constructing and analyzing new families of exceptional orthogonal polynomials.

TS12-3-2

A generalization of DEK-type orthogonal polynomials

Rachel Bailey[1], Otar Tchitchinadze[2]

[1]Bentley University, Massachusetts, USA
[2]University of Connecticut, Connecticut, USA

Email: rbailey@bentley.edu

It is well known that for a family of orthogonal polynomials with respect to a positive measure μ, one can multiply the measure by a polynomial to obtain a new sequence of orthogonal polynomials via the Christoffel transformation. However, in some nonstandard situations, the Christoffel transformation cannot be directly applied. One such situation is when the family of polynomials is a sequence of orthogonal polynomials which is missing finitely many degrees in the sequence. It has already been shown how to modify the Christoffel transformation to one of the first known examples of exceptional orthogonal polynomials introduced by Dubov, Eleonskii and Kulagin, thus, we extend this transformation to a larger class of non-classically orthogonal polynomials. We then explore properties of these polynomials and show their similarities with biorthogonal polynomials and exceptional orthogonal polynomials.

TS13: Multivariate Special Functions

Coordinator: Nicolas Crampé

TS13-1: Day 2 10:30–12:00, KMB 212 (Chair: Margit Rösler)

TS13-1-1

Bivariate polynomials of Tratnik type

Nicolas Crampé[1], Meri Zaimi[2]

[1]CNRS - Université de Tours, Tours, France
[2]Perimeter Institute for Theoretical Physics, Waterloo, Canada

Email: mzaimi@perimeterinstitute.ca

Leonard's theorem provides a classification of the sequences of (univariate) polynomials which satisfy some bispectral properties, namely a three-term recurrence relation and a three-term (q-)difference equation: these are families of orthogonal polynomials which belong to the discrete part of the (q-)Askey scheme. Progress has been made for understanding multivariate generalizations of the (q-)Askey scheme, but there is no multivariate analog of Leonard's theorem. In the spirit of progressing in that direction, I will discuss a certain type of generalized bispectral properties which are related to bivariate polynomials of Tratnik type. These bivariate polynomials are expressed as a product of univariate polynomials with intricate parameters. The goal is to classify all families of bivariate polynomials which satisfy these generalized bispectral properties.

TS13-1-2

Generalized symmetric classical orthogonal polynomials in two variables

Misael E. Marriaga[1], Miguel A. Piñar[2], Gema Alhama[2]

[1]Universidad Rey Juan Carlos, Madrid, Spain
[2]Universidad de Granada, Granada, Spain

Email: misael.marriaga@urjc.es

In this talk I will discuss bivariate symmetric orthogonal polynomials governed by reflection-type interaction weights. In several variables, the geometry of the domain and the symmetry of the weight determine not only the orthogonal basis, but also the natural differential operators acting on it. I will focus on weights with singular interaction factors along reflection walls and on the corresponding orthogonality problems on fundamental chambers.

The main theme is an operator-theoretic construction of these systems. For product domains associated with the classical Hermite, Laguerre and Jacobi weights, for the unit disk through generalized Zernike polynomials, and for the unit triangle with Jacobi-type weights, the symmetric orthogonal families arise as eigenfunctions of self-adjoint second-order differential operators. I will explain how lowering and raising operators shift degrees and parameters, and how their compositions produce additional commuting operators.

The passage to elementary symmetric variables plays a central role: apparent rational singularities disappear and the operators acquire polynomial coefficients. This leads to explicit descriptions of the differential-operator algebras for which the transformed polynomials form a joint eigenbasis, revealing a common Koornwinder-type structure behind these bivariate models with interaction terms.

TS13-1-3

Commutativity, symmetry and Ruijsenaars-type identities

Genki Shibukawa

Kitami Institute of Technology, Hokkaido, Japan

Email: g-shibukawa@mail.kitami-it.ac.jp

We introduce a polynomial $R_{\pm}(u, v; z) \in K(z)[u, v]$, defined as a sum over disjoint subsets $I \sqcup J = L$ of products $A_I(u, z)A_J(v, z \pm \epsilon_J)C_{J,I}(z)C_{I,J}(z \pm \epsilon_J)$, where the functions $A_I(u, z)$ and $C_{I,J}(z)$ satisfy the distribution and reduction laws. For appropriate choices of these functions, the polynomial satisfies the symmetry $R_{\pm}(u, v; z) = R_{\pm}(v, u; z)$, which we call the Ruijsenaars-type identity.

In this talk, we present concrete examples of these identities and two applications: (1) the commutativity of certain difference operators $D(u, x)$ built from $A_I(u, z)$ and $C_{I,J}(z)$, including difference and Pieri operators for interpolation Jack and Macdonald polynomials of types A and BC; (2) a proof of the W-invariance of tableau combinatorial formulas for Okounkov's BC-type interpolation Macdonald polynomials. (Joint work with M. Noumi)

TS13-2: Day 2 15:00–16:30, KMB 212 (Chair: Nicolas Crampé)

TS13-2-1

A generalization of multivariable μ-function and q-Borel summations

Satoshi Tsuchimi[1], Genki Shibukwa[2]

[1]Kindai University, Osaka, Japan
[2]Kitami Institute of Technology, Hokkaido, Japan

Email: tsuchimi@math.kindai.ac.jp

In this talk, we introduce a generalization of Zwegers' multivariable μ-function by iteratively applying the q-Borel summation to a divergent series solution of a q-difference equation. Moreover, we give some formulas of the multivariable generalized μ-function such as shift formulas, symmetries and mock modular properties.

TS13-2-2

Transforms between multivariate Laguerre and Meixner polynomials and their applications to intertwining relations for stochastic processes

Yosuke Kawamoto[1], Genki Shibukawa[2]

[1]Okayama University, Okayama, Japan
[2]Kitami Institute of Technology, Hokkaido, Japan

Email: y-kawamoto@okayama-u.ac.jp

We investigate a close relationship between multivariate Laguerre and Meixner polynomials. A key ingredient is a pair of generating-function identities: one relates a double generating function for multivariate Meixner polynomials to a generating function for Laguerre polynomials, whilst the other relates another double generating function for multivariate Meixner polynomials to a double generating function for Laguerre polynomials. These identities give rise to integral transforms between the two families.

Furthermore, as an application, we derive intertwining relations between Markov processes with continuous and discrete state spaces. An intertwining relation means that two stochastic processes commute through a Markov kernel linking their state spaces. We show that, for general inverse temperature, the Markov process generated by the difference operator whose eigenfunctions are the Meixner polynomials satisfies intertwining relations with the Laguerre process.

TS13-2-3

Uniform bounds on the Dunkl kernel

Lukas Langen

Paderborn University, Paderborn, Germany

Email: llangen@math.upb.de

In Dunkl theory the Dunkl kernel replaces the classical exponential function in its predominant role in harmonic analysis. For regular spectral parameters, we present upper bounds for the Dunkl kernel and its derivatives which are uniform in the spatial variable. These estimates generalize sharp uniform upper bounds for spherical functions of Cartan motion groups and classical Bessel functions. The proof is based on an asymptotic study of the differential system satisfied by the Dunkl kernel with respect to parabolic subgroups of the Weyl group. As a consequence we obtain results regarding the Lebesgue density of the representing measure of Dunkl's intertwining operator.

TS13-3: Day 2 16:45–17:15, KMB 212 (Chair: Nicolas Crampé)

TS13-3-1

Littlewood-Paley theory for orthogonal expansions associated with root systems

Margit Rösler, Lukas Langen

Paderborn University, Paderborn, Germany

Email: roesler@math.upb.de, llangen@math.upb.de

Littlewood-Paley theory is a classical topic of harmonic analysis with many applications, such as singular integral operators or Riesz transforms. For compact Lie groups, Littlewood-Paley theory was developed by Stein in the 1970ies. More recently, there have been various extensions to rational Dunkl theory. In the present talk, we establish basic concepts of Littlewood-Paley theory for expansions in terms of Heckman-Opdam polynomials, which generalize both classical Jacobi polynomials as well as the spherical functions of compact symmetric spaces.

In the spirit of Stein's work for compact Lie groups, we study Littlewood-Paley g-functions involving Cherednik operators and the Heckman-Opdam Laplacian. As an application, natural bounds on Riesz transforms in this setting are obtained.

This session will be continued in TS03 (17:15–18:15).

TS14: Representation Theory and Special Functions

Coordinator: Luc Vinet

TS14: Day 2 10:30–12:00, Clover Hall (Chair: Luc Vinet)

TS14-1-1

A q-symmetrization Hamiltonian and a family of orthogonal idempotents

Luc Lapointe[1], Luc Vinet[2]

[1]Universidad de Talca, Talca, Chile
[2]Université de Montréal, Montréal, Canada

Email: llapointe@utalca.cl, luc.vinet@umontreal.ca

We define an action of the 0-Hecke algebra, which is generated by $\pi_1, \ldots, \pi_{m-1}$, on the $\mathbb{C}(q)$-vector space V_m of dimension $m!$ with basis $\{|\boldsymbol{a}\rangle \mid \boldsymbol{a}$ is a permutation of $(1, \ldots, m)\}$. The action of π_i amounts to a symmetrization of the entries i and $i+1$ up to a factor in $\mathbb{C}(q)$:

$$\pi_i |ab\rangle = \frac{q^a}{q^a - q^b}\big(|ab\rangle + |ba\rangle\big).$$

In the 0-Hecke algebra, we will consider in particular the Hamiltonian

$$H_q = \pi_1 + \cdots + \pi_{m-1},$$

which is a sum of q-symmetrization operators. Even though H_q is not Hermitian or diagonalizable, it has real eigenvalues and we will show how to use a family of orthogonal idempotents $\{L_w\}_{w \in S_m}$ to construct a basis of V_m on which H_q acts triangularly. As a consequence, we can obtain the time evolution

$$e^{tH_q} L_w |1, \ldots, m\rangle$$

of any state in our basis. Finally, we will connect the Hamiltonian H_q to the Hamiltonian of the Heisenberg model.

TS14-1-2

Gaudin models for some classical multivariate distributions

Plamen Iliev

Georgia Institute of Technology, Atlanta, USA

Email: iliev@math.gatech.edu

In the analysis of the generic quantum superintegrable system on the sphere [1] it was discovered that, appropriately normalized, the integrals of motion define a representation of the Kohno–Drinfeld Lie algebra with self-adjoint operators with respect to an inner product induced by the Dirichlet distribution. This was extended to several discrete multivariate distributions in [2]. In my talk, I will briefly review the diagonalization of the corresponding Gaudin operators for the multinomial distribution [3], and then discuss the Gaudin models for the multivariate Hahn and Dirichlet distributions [4].

References

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[4] P. Iliev, *Gaudin models for some classical multivariate distributions*, in preparation.

TS14-1-3

Proof of the Kresch-Tamvakis conjecture

Taiyo S. Terada, John S. Caughman

Portland State University, Portland, USA

Email: taiyoterada@gmail.com

In a recent article in the Proc. Amer. Math. Soc., we proved a conjecture of Kresch & Tamvakis from 2001 about a certain ${}_4F_3$ hypergeometric series. In this talk, we outline our proof of the conjecture.

TS15: General Orthogonal Polynomials, Hypergeometric and Special Functions (including Mathieu, Lamé, Heun, etc.)

Coordinator: Kouichi Takemura

TS15-1: Day 4 10:30–12:00, Clover Hall (Chair: Rakesh K. Parmar)

TS15-1-1

Hypergeometric and q-hypergeometric polynomials with finite free convolutions

Andrei Martínez-Finkelshtein[1,2], Rafael Morales[1], Daniel Perales[3]

[1]Baylor University, TX, USA
[2]University of Almería, Spain
[3]University of Notre Dame, IN, USA

Email: a_martinez-finkelshtein@baylor.edu

Tools from free probability theory–most notably finite free convolutions of polynomials–have proved particularly useful in the study of hypergeometric polynomials. They make it possible to represent multi-parameter families of hypergeometric polynomials as finite free convolutions of simpler elementary building blocks. Combined with the preservation of real-rootedness and interlacing under these convolutions, such representations provide an effective framework for determining when a given hypergeometric polynomial has only real zeros, as well as for studying the monotonicity of its zeros with respect to parameters.

The same perspective also extends naturally to asymptotic questions: the limiting zero distributions of generalized hypergeometric polynomials can often be expressed as free convolutions of simpler limiting measures.

Recently, these ideas have been extended to q-hypergeometric polynomials. This requires adapting the notion of convolution to the q-setting, but leads to analogous structural representations of multi-parameter families in terms of elementary factors. These representations, in turn, provide new tools for analyzing zero location, real-rootedness, and interlacing properties for q-hypergeometric polynomials.

TS15-1-2

q-Polynomial perturbations of Heine's and Jackson's transformations

Dmitrii Karp[1], Thu Hien Nguyen[2]

[1]Holon Institute of Technology, Holon, Israel
[2]Julius-Maximilian University of Würzburg, Würzburg, Germany

Email: dimkrp@gmail.com

Polynomial perturbation of a hypergeometric series is obtained by multiplying its coefficients by a fixed polynomial evaluated at the summation index. Polynomial perturbations of Euler's and Pfaff's transformations, i.e. transformations of the polynomially perturbed Gauss ${}_2F_1$ series, are known as Miller-Paris transformations. Transformations of the basic hypergeometric functions ${}_2\phi_1$ due to Heine and Jackson represent q-analogues of Euler's and Pfaff's transformations. In the talk we discuss their generalizations to q-analogues of Miller-Paris transformations. To this end we consider q-polynomial perturbations which amount to multiplication of power series coefficients by values of a fixed polynomial at non-negative powers of q. It turns out that there are at least three q-analogues of two Miller-Paris transformations. We relate them to the q-analogue of the Karlsson-Minton summation theorem due to Gasper and other summation formulas for the basic hypergeometric functions with q-deformed integral parameter differences. We further discuss the connection with q-interpolation (interpolation with nodes at powers of q) and combinatorial inversion formulas.

TS15-1-3

An affine Weyl group action on the basic hypergeometric series arising from the cluster algebra

Takao Suzuki

Kindai University, Osaka, Japan

Email: suzuki@math.kindai.ac.jp

Recently, a cluster algebraic construction of affine Weyl groups was established by Masuda, Okubo and Tsuda. The obtained birational representations provide several discrete Painlevé equations and their generalizations as translations. In particular, the q-Garnier system is derived from the affine Weyl group of type $(A_{2n+1} + A_1 + A_1)^{(1)}$. The q-Garnier system was first proposed by Sakai as a deformation problem of a linear q-difference equation. Afterward, Nagao and Yamada considered other deformations of the same linear q-difference equation and presented several systems of nonlinear q-difference equations (called other directions of the q-Garnier system) with the aid of the Padé method. The derivation of all directions from the affine Weyl group was given by Okubo and the speaker.

On the other hand, the q-Garnier system admits a particular solution in terms of the basic hypergeometric series ${}_{n+1}\phi_n$ (or equivalently the q-Lauricella series $\phi_D^{(n)}$). Hence it becomes a next problem to investigate the action of the affine Weyl group on the q-hypergeometric series. In this talk we give an answer to this problem. We realize an affine Weyl group of type $(A_n + A_n)^{(1)}$ as left actions of $(n+1)\times(n+1)$ matrices on an $(n+1)$-dimensional vector whose components are described in terms of ${}_{n+1}\phi_n$. Then the linear q-difference equation and the q-contiguity relations for ${}_{n+1}\phi_n$ are derived as translations. Besides, we obtain a particular solution satisfying several directions of the q-Garnier system simultaneously.

This talk is based on the collaboration with Taiki Idomoto.

TS15-2: Day 4 15:00–16:30, Clover Hall (Chair: Taikei Fujii)

TS15-2-1

Confluent q-Heun equations based on Newton polygon

Chihiro Sato, Kouichi Takemura

Ochanomizu University, Tokyo, Japan

Email: clover.chihiro@icloud.com

The q-Heun equation is a q-difference analogue of the Heun equation, written as below.

$$(\alpha_2 x^2 + \alpha_1 x + \alpha_0) f(qx) - (\beta_2 x^2 + \beta_1 x + \beta_0) f(x) + (\gamma_2 x^2 + \gamma_1 x + \gamma_0) f(x/q) = 0, \ \alpha_2\alpha_0\gamma_2\gamma_0 \neq 0.$$

Also, as with the Heun equation and the Painlevé equation, the q-Heun equation is related to a q-analogue of the sixth Painlevé equation. Based on this relation and degenerations of the q-Painlevé equation in [KNY, M], we introduced some confluent q-Heun equation s in [ST].

Here, we consider singularity on $x = 0, \infty$ because of q-shift operator's behavior. Those regularity is characterized by shapes of Newton polygon. We research Newton polygons of the above and other confluent q-Heun equations to further research for solutions of the q-Heun equation and its degenerations.

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TS15-2-2

q-integral transformation of q-Heun equation and its applications

Ayaka Murakami, Kouichi Takemura

Ochanomizu University, Tokyo, Japan

Email: ayaka.murakami.math@gmail.com, takemura.kouichi@ocha.ac.jp

Heun's differential equation is the standard form of a second order Fuchsian linear ordinary differential equation with four regular singularities $\{0, 1, t, \infty\}$ under the condition $\gamma + \delta + \epsilon = \alpha + \beta + 1$. Heun's differential equation contains an accessory parameter. Hahn [1] introduced the q-Heun equation, which is a q-difference analogue of Heun's differential equation of the form

$$\{a_2x^2 + a_1x + a_0\}g(x/q) - \{b_2x^2 + b_1x + b_0\}g(x) + \{c_2x^2 + c_1x + c_0\}g(xq) = 0,$$

where $a_2a_0c_2c_0 \neq 0$. The q-Heun equation was rediscovered in [2] by considering degeneration of the Ruijsenaars–van Diejen system four times. Takemura [3] showed a theorem of q-integral transformation for the q-Heun equation and, by applying it, investigated q-integral transformations of monomial-type solutions of the q-Heun equation. In this talk, we present q-integral transformations of polynomial-type solutions of the q-Heun equation, which are generalizations of the monomial-type solutions. In particular, by specializing certain polynomial-type q-integral solutions, we are able to express them as finite sums of q-hypergeometric series.

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TS15-2-3

On zeros of some ${}_2\phi_1$ polynomials associated with q-Meixner polynomials

Priyabrat Gochhayat[1], Pinaki Prasad Kar[2]

[1]Sambalpur University, Odisha, India
[2]Siksha 'O' Anusandhan University, Odisha, India

Email: pgochhayat@gmail.com

In this talk, we consider a class of ${}_2\phi_1$ basic hypergeometric polynomials associated with the q-Meixner polynomials. We focus on the study of the distribution of their zeros. Using the generalized form of Markov's monotonicity theorem together with suitable q-contiguous relations satisfied by these polynomials, we establish several interlacing properties for their zeros. In particular, we show that the zeros of different sequences of ${}_2\phi_1$ polynomials satisfy Stieltjes-type interlacing. We also present new bounds for both the smallest and largest zeros of these polynomials. These results provide further insight into the structure and behavior of zeros of basic hypergeometric polynomials.

TS15-3: Day 4 16:45–18:15, Clover Hall (Chair: Dmitrii Karp)

TS15-3-1

Linear relations for Kajihara's q-hypergeometric series

Taikei Fujii[1], Takahiko Nobukawa[2]

[1]Ochanomizu University, Tokyo, Japan
[2]Kogakkan University, Mie, Japan

Email: fujii.taikei@ocha.ac.jp

Kajihara's q-hypergeometric series $W^{M,N}$ [Kajihara, 2004, Adv. Math.] is a multivariable generalization of the very well-poised q-hypergeometric series ${}_{r+1}W_r$. The series $W^{M,N}$ is an extension of Holman's hypergeometric series and Milne's q-hypergeometric series. The series $W^{M,N}$ has a duality transformation formula. One of the background for series $W^{M,N}$ is Macdonald's theory, e.g., the duality transformation formula is derived from a kernel identity for Macdonald's q-difference operators. This duality formula is applied widely in the theory of integrable systems and mathematical physics. On the other hand, no q-difference equations satisfied by $W^{M,N}$ has been well known.

The variant of Heine's q-hypergeometric equation of degree three [Hatano–Matsunawa–Sato–Takemura, 2022, Funk. Ekvac.] was introduced from the viewpoint of quantum integrable systems. Solutions in terms of Euler type Jackson integral and the very well-poised q-hypergeometric series ${}_8W_7$ were studied by the authors [arXiv:2207.12777.] and Arai–Takemura [2023, SIGMA]. Considering a generalization of this integral solution, a higher order extension E_M was introduced recently by the second author [2025, Ramanujan J.]. It was also found that Kajihara's q-hypergeometric series $W^{M,2}$ (of the case $N = 2$) satisfies this equation E_M.

In this talk, we give a solution of connection problem for the equation E_M. First, connection formulas for integral solutions of E_M are discussed. Second, we give new linear relations for Kajihara's q-hypergeometric series $W^{M,2}$ using these connection formulas. This talk is based on the paper [arXiv:2404.00969.].

TS15-3-2

Extended Exton's double hypergeometric functions: integral forms and functional bounds

Rakesh K. Parmar

Pondicherry University (A Central University), Puducherry, India

Email: rakeshparmar27@gmail.com

In this present work, we aim to extend and systematically investigate the some particular Exton's double hypergeometric function $X^{A:B;B'}_{C:D;D'}[x, y]$, which is motivated by the recent integrated version of the Euler's Beta integral form with a MacDonald function $K_\nu(z)$ in the integrand. The newly introduced extended Exton's double hypergeometric functions $X^{A:B;B'}_{C:D;D'}[x, y; p, q, \nu, \lambda]$ is then represented by a number of integral representations of the Euler and Laplace types, including several further representations involving Bessel $J_\nu(z)$ and modified Bessel functions $I_\nu(z)$ of the first kind along with recurrence formulae. Using existing functional bounds for extended Euler's Beta function, various functional upper bounds are derived for particular extended Exton's double hypergeometric functions $X^{A:B;B'}_{C:D;D'}[x, y; p, q, \nu, \lambda]$. Also, several bounding inequalities are derived by virtue of Luke's, von Lommel's, Minakshisundaram and Szász and Olenko bounds. Further, with a newly introduced probability distribution applying extended Kummer and of Horn functions, for which moment inequalities of Turán type are proved.

TS15-3-3

Integral representations and functional bounds for the generalized Horn's function

Rakesh K. Parmar[1], Tibor K. Pogány[2,3], Pirivina S[1]

[1]Pondicherry University (A Central University), Puducherry, India
[2]University of Rijeka, Rijeka, Croatia
[3]Óbuda University, Budapest, Hungary

Email: pirivina1610@pondiuni.ac.in, pirivina1610@gmail.com

We aim to generalize Horn's double hypergeometric function $H_4[x, y]$, inspired by the recent unified version of Euler's Beta integral form with a MacDonald function in the integrand. Next, we establish integral representations of the Laplace and Euler types for the generalized Horn's double hypergeometric function, along with some additional representations involving Bessel $J_\nu(z)$ and Modified Bessel functions $I_\nu(z)$. Functional bounds of the extended Euler's Beta function are used to generate many functional upper bounds for the generalized Horn's double hypergeometric function $H^{\lambda}_{4,p,q,\nu}$ including extended Gaussian hypergeometric $F^{\lambda}_{p,q,\nu}[z]$ and the extended Kummer's confluent hypergeometric $\Phi^{\lambda}_{p,q,\nu}[z]$. Luke's, von Lommel's, Minakshisundaram, and Szász and Olenko bounds are used to obtain several further bounding inequalities.

TS15-4: Day 5 10:30–12:00, Clover Hall (Chair: Takao Suzuki)

TS15-4-1

Asymptotic analysis of the confluent Dotsenko–Fateev equation

Shunya Adachi

Utsunomiya University, Tochigi, Japan

Email: sadachi@a.utsunomiya-u.ac.jp

The *Dotsenko–Fateev integral*

$$U(z) = \int_{0\le t_2\le t_1\le 1} (t_1 - t_2)^g \prod_{i=1}^{2} t_i^a (t_i - 1)^b (t_i - z)^c \, dt_1 dt_2$$

was introduced in the study of two-dimensional conformal field theory as a hypergeometric-type integral representation of certain four-point conformal blocks. Here, a, b, c, and g are complex parameters. Dotsenko and Fateev (1984) showed that this integral satisfies a third-order linear Fuchsian ordinary differential equation with respect to z, now known as the *Dotsenko–Fateev equation*. This equation can be regarded as a higher-order analogue of the Gauss hypergeometric differential equation. Accordingly, several aspects of its global analytic structure, such as the connection problem, the monodromy representation, and irreducibility conditions, have been studied by several authors, including Dotsenko and Fateev (1984) and Mimachi (2013, 2024).

In this talk, we study a confluent version of the Dotsenko–Fateev equation, which has one regular singular point and one irregular singular point. We call it the *confluent Dotsenko–Fateev equation*. This equation can be regarded as a higher-order analogue of Kummer's confluent hypergeometric equation. Our main result is an explicit computation of the Stokes matrices at the irregular singular point via integral representations; their entries are expressed in terms of Gamma functions of the parameters. Time permitting, we will discuss how applying the middle Laplace transform to the confluent Dotsenko–Fateev equation yields certain special cases of the Heun equation.

TS15-4-2

Irregular complex hypergeometric integrals

Yoshiaki Goto[1], Saiei-Jaeyeong Matsubara-Heo[2]

[1]Otaru Commerce University, Otaru, Japan
[2]Tohoku University, Sendai, Japan

Email: goto@res.otaru-uc.ac.jp, saiei@tohoku.ac.jp

The complex hypergeometric integral is an analogue of the Euler integral representation of the Gauss hypergeometric function ${}_2F_1$ where the integration kernel contains absolute values and the integration domain is the whole complex plane. It was repeatedly re-discovered in many contexts such as conformal field theory. In [2], a generalization of complex hypergeometric integrals to generalized hypergeometric function ${}_pF_q$ was studied, where it is established that the complex hypergeometric integral is a quadratic combination of the classical ${}_pF_q$. This includes the case $p < q + 1$, thus the differential equation for ${}_pF_q$ develops irregular singularity.

In this talk, we show that a certain class of complex hypergeometric integrals of several variables is expanded into a quadratic combination of Gel'fand-Kapranov-Zelevinsky hypergeometric functions, providing a vast generalization of the result in [2]. The technique we employ can be traced back to earlier work by Aomoto [1]. We re-interpret the formula as a Riemann-Hodge bilinear relation of a twisted rapid decay (co)homology.

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TS15-4-3

Matrix equations and orthogonal polynomials

Richard Mikaël Slevinsky, Gabriel Hamm, Karim Gumerov, Samantha Rigg

University of Manitoba, Canada

Email: richard.slevinsky@umanitoba.ca

It is well known that matrices with low Hessenberg-structured displacement rank enjoy fast algorithms for certain matrix factorizations. We show how $n \times n$ principal finite sections of the Gram matrix for the orthogonal polynomial measure modification problem have such a displacement structure, unlocking a collection of fast algorithms for computing connection coefficients (as the upper-triangular Cholesky factor) between a known orthogonal polynomial family and the modified family. In general, the $O(n^3)$ complexity is reduced to $O(n^2)$, and if the symmetric Gram matrix has upper and lower bandwidth b, then the $O(b^2n)$ complexity for a banded Cholesky factorization is reduced $O(bn)$. Next, we will extend this basic result to a matrix equation for the Sobolev–Gram matrix which has a number of terms proportional to the order of the corresponding Sobolev inner product. Finally, we will discuss the structure of Gram matrices of multivariate orthogonal polynomials. These multivariate Gram matrices are endowed with a block structure, and they satisfy a number of simultaneous matrix equations equal to the dimension of the domain.

TS15-5: Day 5 15:00–16:00, Clover Hall (Chair: Shunya Adachi)

TS15-5-1

Single-point completed interlacing for orthogonal and non-orthogonal polynomial families

Kerstin Jordaan, Vikash Kumar

University of South Africa, Pretoria, South Africa

Email: jordakh@unisa.ac.za

The study of interlacing zeros from different polynomial sequences goes back at least to Levit's work on Hahn polynomials and Askey's results on shifted Jacobi polynomials. These results show that interlacing phenomena are not confined to consecutive polynomials in a single orthogonal sequence, but may also arise when parameters or polynomial families are changed.

In this talk, I will consider situations in which two polynomials of the same or consecutive degree from related families have real zeros which do not interlace. We show that, in several such cases, the missing alternation can be restored by adding a single point which arises naturally as the zero of a linear coefficient in a mixed recurrence relation yielding what we call a single-point completed interlacing pattern. This context is different from the classical Stieltjes-type problem, where the degree difference between the two polynomials is higher and additional points are needed to complete interlacing because one polynomial has too few zeros.

Our approach provides a consolidated framework which does not rely on orthogonality and is broadly applicable to both orthogonal and non-orthogonal polynomials which fail to interlace by exactly one point. We illustrate this with new interlacing results for zeros of Krawtchouk, Meixner and Narayana polynomials. We also show that the general framework can be used to recover, refine and extend known results on completed interlacing for Laguerre and Jacobi polynomials from different sequences.

TS15-5-2

Two-point completed interlacing and cross-family Wronskian inequalities for related polynomial sequences

Vikash Kumar, Kerstin Jordaan

University of South Africa, Pretoria, South Africa

Email: vikaskr0006@gmail.com

We study completed interlacing of zeros for pairs of polynomials from different sequences whose zeros fail to interlace by exactly two points. Using a general mixed recurrence relation, we identify a quadratic polynomial whose zeros provide the two additional points needed to complete the interlacing, and determine their positions relative to the zeros of the higher-degree polynomial. We apply this framework to classical Jacobi polynomials, improving an earlier result for $P_n^{(\alpha,\beta)}$ and $P_{n+1}^{(\alpha+1,\beta+1)}$ by explicitly identifying the extra interlacing points. As an application of the general completed interlacing result, we derive a cross-family Wronskian inequality, together with a sharpened form, for related polynomial sequences. Its application to Jacobi polynomials yields a doubly parameter-shifted Turán-type inequality involving $P_n^{(\alpha,\beta)}$, $P_n^{(\alpha+2,\beta+2)}$, and $P_{n\pm1}^{(\alpha+1,\beta+1)}$. We also present related completed interlacing results arising from a different mixed recurrence relation, with applications to Pseudo-Jacobi and Meixner-Pollaczek polynomials. In particular, we address an open question for Meixner–Pollaczek polynomials with a unit parameter shift.

TS16: Applications of Orthogonal Polynomials and Special Functions – Interdisciplinary Topics (including Numerical Analysis, Data Science, Machine Learning, etc.)

Coordinators: Satoshi Tsujimoto, Hiroshi Miki

TS16-1: Day 2 15:00–16:30, Clover Hall (Chair: Lidia Fernández)

TS16-1-1

On g-orthogonal polynomials: a theoretical tool arising from applications

Iván Area

Universidade de Vigo, Spain

Email: area@uvigo.gal

In a recent work [1] we have considered a mathematical model based on Stieltjes differential equations [2] to analyze the spread of the invasive species (in Europe) *Vespa Velutina*. Stieltjes derivatives, or g-derivatives, generalize time scales (and thus also difference equations and equations with impulses) [3]. In this framework, a new class of special functions is introduced, by combining the very classical theory of univariate orthogonal polynomials with the theory of Stieltjes derivatives. These functions are expressed both in terms of classical monomials and also in terms of g-monomials. An example is treated in detail, providing explicit expression for the coefficients of the three-term recurrence relation as well as of the second order differential equation they satisfy when expressed in terms of classical monomials and in terms of g-monomials. This is a joint work with Francisco J. Fernández, Mohammadali Jafary and F. Adrián F. Tojo.

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TS16-1-2

New families of the parametric Apostol polynomials and generalized Voigt Functions

Nabiullah Khan

Aligarh Muslim University, Aligarh, India

Email: nukhanmath@gmail.com

Over the last several years, a number of authors have introduced and investigated several types of the parametric polynomials (for example, parametric Bernoulli and Apostol-Bernoulli polynomials, parametric Euler and Apostol-Euler polynomials, parametric Genocchi and Apostol-Genocchi polynomials etc) by making use of the exponential and trigonometric functions. In a sequel, in this paper, we present a new family of the parametric Apostol-Bernoulli, Apostol-Euler, Apostol-Genocchi and Fubini type polynomials by utilizing the Bessel function of the first kind. We also investigate some fundamental properties of our introduced polynomials. Moreover, few applications of our defined parametric polynomials with the well-known generalized Voigt functions are pointed out.

TS16-1-3

The singular values of Lévy's area matrix

Danilo Jr Dela Cruz, Harald Oberhauser

University of Oxford, Oxford, UK

Email: delacruz@maths.ox.ac.uk

The matrix of Lévy's areas $\mathbf{A}_t$ of d-dimensional Brownian motion B_t is a fundamental object in stochastic analysis and generating samples of $(B_t, \mathbf{A}_t)$ is crucial to generating strong approximations to solutions of SDEs. By deriving an explicit formula for the density of $\mathbf{A}_t$, we show its singular values form a biorthogonal ensemble and standard DPP-machinery allows us to generate exact samples of $\mathbf{A}_t$. The joint density of $(B_t, \mathbf{A}_t)$ is more complicated and bottlenecked by a Harish-Chandra-like integral which we approach with Weingarten Calculus.

TS16-2: Day 2 16:45–18:15, Clover Hall (Chair: Nicola Mastronardi & Satoshi Tsujimoto)

TS16-2-1

Approximation of complex-valued functions by rational interpolation operators: theory and applications

Priyanka Majethiya, Shivam Bajpeyi

Sardar Vallabhbhai National Institute of Technology Surat, Gujarat, India

Email: priyankamajethiya2000@gmail.com, shivambajpai1010@gmail.com

The approximation of complex-valued functions is of fundamental importance as it provides a rigorous framework for amplitude and phase-dependent phenomena. In this paper, we propose a family of complex rational interpolation operators to approximate analytic as well as non-analytic functions along with real-life application. In this direction, the first operator is constructed using Chebyshev polynomials of the first kind, namely classical complex rational interpolation operators for approximating non-analytic functions. We establish approximation results for these operators utilizing the notion of the modulus of continuity. To approximate non-analytic but integrable function, we define Kantorovich type complex rational interpolation operators and establish their boundedness and convergence. Furthermore, in order to approximate functions preserving higher derivatives, we introduce Hermite type complex rational interpolation operators and study their approximation capabilities using higher order of modulus of continuity. To validate the theoretical results, we provide numerical simulations illustrating approximation ability of proposed family of complex rational interpolation operators. Moreover, we present the utility of our construction in denoising of signals corrupted from Gaussian-noise. In addition, we demonstrate its real-life application in brain image reconstruction by applying these operators to real brain imaging data, particularly magnetic resonance imaging (MRI) dataset. The algorithms are provided at the end to formally explain the reconstruction procedure of the proposed family of complex rational interpolation operators.

TS16-2-2

Approximation properties of fractional Szász-Kantorovich operators involving Hermite polynomials

Manoj Kumar[1], M. Mursaleen[2]

[1]SR University, Warangal, India
[2]Aligarh Muslim University, Aligarh, India

Email: kmanoj665@yahoo.com

The primary objective of this article is to introduce a sequence of positive linear operators constructed via the Riemann–Liouville fractional integral and Hermite polynomials. Several estimates involving test functions and central moments are established to derive the rate of convergence and the order of approximation. Furthermore, uniform convergence is established using a Korovkin-type theorem, while quantitative estimates are given in terms of the first modulus of smoothness. Approximation results are also investigated through Peetre's K-functional, the second modulus of continuity, and Lipschitz-type function spaces. The weighted approximation properties of these operators are further established in terms of the modulus of continuity. The analysis is subsequently extended to a two-dimensional setting, where the corresponding approximation properties are rigorously examined. To support the theoretical developments, numerical examples are presented that illustrate the effectiveness of the proposed operators and confirm the theoretical error estimates. Finally, concluding remarks and possible directions for future research are provided.

TS16-2-3

Product integration rules for highly oscillatory integrals

Nicola Mastronardi, Teresa Laudadio

Istituto per le Applicazioni del Calcolo, Cosiglio Nazionale delle Ricerche, Bari, Italy

Email: nicola.mastronardi@cnr.it

Given the integrals

$$I(\cos, f) = \int_0^\infty x^\alpha e^{-\beta x}\cos(\omega x) f(x)dx, \quad I(\sin, f) = \int_0^\infty x^\alpha e^{-\beta x}\sin(\omega x) f(x)dx, \qquad \alpha > -1, \ \ \beta > 0, \ \ \omega > 0,$$

product–integration rules are considered for their computation:

$$I_P(\cos, f) = \sum_{k=1}^{n} w_{n,k}^{(\cos)} f(x_{n,k}), \quad I_P(\sin, f) = \sum_{k=1}^{n} w_{n,k}^{(\sin)} f(x_{n,k}),$$

with

$$w_{n,k}^{(\cos)} = w_{n,k}\sum_{j=0}^{n-1}\mathcal{L}_j(x_{n,k})\mathcal{M}_j, \quad w_{n,k}^{(\sin)} = w_{n,k}\sum_{j=0}^{n-1}\mathcal{L}_j(x_{n,k})\mathcal{N}_j, \quad , k = 1, \dots, n,$$

where $x_{n,k}$ and $w_{n,k}$ are the nodes and weights of the n–point Gauss–Laguerre quadrature rule, $\mathcal{L}_j(x), \ j = 0, 1, \dots,$ are the Laguerre polynomials, and

$$\mathcal{M}_j = \int_0^\infty x^\alpha e^{-\beta x}\cos(\omega x)\mathcal{L}_j(x)dx, \quad \mathcal{M}_j = \int_0^\infty x^\alpha e^{-\beta x}\sin(\omega x)\mathcal{L}_j(x)dx, \qquad j = 0, 1, 2, \dots,$$

are the *modified moments*. Computational issues are discussed and some numerical tests are reported, showing the reliability and the efficiency of the proposed method.

TS16-3: Day 3 10:30–11:30, Clover Hall (Chair: Iván Area)

TS16-3-1

UL and LU Factorizations of birth-death processes associated with classical orthogonal polynomials

Jose Arcia Manoleskos[1,2], Manuel Domínguez de la Iglesia[3]

[1]Universidad de Sevilla, Sevilla, Spain
[2]Universidad de Panamá, Panama
[3]Universidad de Alcalá, Alcalá de Henares, Spain

Email: josarcman@alum.us.es

In this work, we study upper-lower (UL) and lower-upper (LU) factorizations of the infinitesimal operator associated with a continuous-time birth-death process. Our focus is on those factorizations whose Darboux transformations are again the infinitesimal operator of some birth-death process. For both types of factorization, we establish necessary and sufficient conditions for their existence and provide a probabilistic interpretation of the entries of the resulting factors. Finally, we apply these results to birth-death processes whose infinitesimal operators are associated with classical families of orthogonal polynomials, namely Chebyshev, Charlier, Meixner, and Laguerre polynomials. For each case, we determine the recurrence of the transformed processes by analyzing the corresponding transformed spectral measure.

TS16-3-2

Approximation and cubature on planar domains via Zernike-like bases

Ruymán Cruz-Barroso[1], Lidia Fernández[2], Francisco Marcellán[3], Juan Antonio Villegas[2]

[1]La Laguna University, Spain
[2]Universidad de Granada, Granada, Spain
[3]Universidad Carlos III de Madrid, Spain

Email: lidiafr@ugr.es

We consider the problem of approximating and numerically integrating functions defined on specific planar domains, namely ellipses, circular annuli, and regular polygons. The function is assumed to be known at a finite set of sample points; a subset of these points is used to impose interpolation conditions, while the remaining ones are exploited to improve the approximation through a regression step. Orthogonal systems are constructed by transporting the classical Zernike basis on the unit disk to each of the considered geometries by means of suitable smooth bijections, leading to Zernike-like bases adapted to the underlying domain. This approach yields stable approximation operators and effective cubature formulas, obtained via a change of variables from Gaussian integration rules on the disk. Numerical experiments on ellipses, annuli, and regular polygons illustrate the accuracy and robustness of the method, as well as its practical efficiency for approximation and numerical integration on these planar domains.

Posters (Day 3, 12:00–14:30, B1F Lobby)

PS01-1 (Painlevé and Integrable Systems)

Construction of $\mathbb{Z}_2^2$-integrable hierarchy using $\mathbb{Z}_2^2$-graded algebras

Naruhiko Aizawa, Ichi Fujii, Ren Ito

Osaka Metropolitan University, Osaka, Japan

Email: sq25482d@st.omu.ac.jp

It is known that hierarchies of integrable equations, including the Liouville, sinh-Gordon, and modified KdV (mKdV) equations, can be constructed from the loop algebra of the Lie algebra $\mathfrak{sl}(2)$. These equations play a crucial role in soliton theory and field theory. The objective of this research is to extend this construction to $\mathbb{Z}_2^2$-graded algebras, thereby introducing novel integrable systems. Through this extension, we can obtain soliton equations that incorporate exotic fields in addition to standard bosonic fields. We have successfully carried out this extension for the case of the loop algebra of $\mathbb{Z}_2^2$-$\mathfrak{osp}(1|2)$, which contains $\mathfrak{sl}(2)$ as a subalgebra, and successfully obtained the integrable $\mathbb{Z}_2^2$-graded extensions of the aforementioned equations.

PS01-2 (Painlevé and Integrable Systems)

Construction and classification of multi-lump solutions of the elliptic two-dimensional Toda lattice equation

Ryosuke Naganuma[1], Yuta Tanaka[2], Ken-ichi Maruno[1], Sarbarish Chakravarty[3]

[1]Waseda University, Tokyo, Japan
[2]Independent researcher, Tokyo, Japan
[3]University of Colorado, Colorado Springs, United States of America

Email: r.naganuma.39@fuji.waseda.jp

In addition to soliton solutions, certain integrable equations possess nonsingular rational solutions known as multi-lump solutions, which form two-dimensional localized pulses. While the multi-lump solutions and the underlying mathematical structure of the Kadomtsev–Petviashvili equation have been extensively studied, those of the elliptic two-dimensional Toda lattice equation have yet to be constructed. In this presentation, the construction of multi-lump solutions for the elliptic two-dimensional Toda lattice equation and their classification using Young diagrams are discussed. First, it is shown that these solutions can be constructed from the Gramian solutions of the elliptic two-dimensional Toda lattice equation with generalized Schur polynomials. It is also shown that each solution corresponds to a specific Young diagram, with the number of localized pulses exactly matching the number of boxes within the diagram. Furthermore, specific solutions related to the special polynomials which appear in the construction of rational solutions of Painlevé equations are investigated. Finally, the continuous limits of these multi-lump solutions are discussed.

PS01-3 (Painlevé and Integrable Systems)

On the quantization of higher order Painlevé systems

Yuichi Ueno

Kogakkan University, Mie, Japan

Email: y-ueno@kogakkan-u.ac.jp

In a series of works around 1980's, Okamoto revealed the Hamiltonian structures of the Painlevé equations and showed that there are affine Weyl group symmetries acting as the Bäcklund transformations. There are several higher order extensions of Painlevé systems (the Garnier system [Garnier (1912), Sasano system [Sasano (2006)], Fuji-Suzuki-Tsuda system [Fuji-Suzuki (2010, 2012), Tsuda (2012, 2014)] including their degenerations) and their polynomial Hamiltonian structures are also well-known. Nagoya constructed quantum versions of Painlevé equations P_{J} ($\mathrm{J} = \mathrm{II}, \cdots, \mathrm{VI}$) which preserve the affine Weyl group symmetries of the classical equations [Nagoya (2004, 2007, 2012)].

For all Painlevé equations, Okamoto constructed the space of initial conditions which parametrize the solutions of P_{J} [Okamoto (1979)]. Based on this result, Takano and his collaborators characterized the Painlevé equations P_{J} ($\mathrm{J} = \mathrm{II}, \cdots, \mathrm{VI}$) by a geometric method [Matano, Matumiya, Shioda and Takano (1997, 1999, 2000)]. Namely, the Painlevé equations written by polynomial Hamiltonians $H_{\mathrm{J}}(q, p)$ are transformed again into holomorphic Hamiltonian systems under certain birational canonical transformations. Furthermore, the Painlevé equations can be uniquely characterized by this holomorphic property.

For cases of Painlevé equations and Garnier systems in two variables, we constructed the quantization of these systems in [Ueno (2009, 2025)] by considering their holomorphic properties (so called *quantum Takano's theory*). In this poster, we will explain quantum Takano's theory for higher order Painlevé systems . We show that the quantum higher order Painlevé systems are uniquely determined by the holomorphic condition.

PS02-1 (Random Matrices and Probability)

Provable Weak-to-Strong generalization for neural networks using random matrix theory

Deborah Oliveira, Elliot Paquette

McGill University, Montreal, Canada

Email: deborah.deoliveira@mail.mcgill.ca

In this work we prove the occurrence of the Weak-to-Strong generalization phenomenon, where a strong model learns from labels generated by a weak model and still outperforms it. We study this phenomenon in a two-layer random features model, where model strength is determined by the number of neurons. Using tools from random matrix theory, in particular deterministic equivalents obtained through a self-consistent Dyson equation, we characterize the population errors of the weak teacher and the strong student. This allows us to find the exact exponent characterizing how the strong model's population error improves relative to the weak model's error for linear and ReLU activation functions and polynomial target functions.

PS04-1 (Orthogonal Polynomials, Mathematical Physics, and Interacting Particle Systems)

3-dimentional spin chain model with nearest neighbor couplings derived from 3-variable Krawtchouk polynomials

Subaru Hamada, Hiroshi Miki

Doshisha University, Kyoto, Japan

Email: ctwn0904@mail4.doshisha.ac.jp

Quantum state transfer is an important task in quantum computation and the *XX* spin chain is a mathematical model describing it. In [1], the 3-dimensional spin chain model was proposed by using 3-variable Krawtchouk polynomials, where perfect state transfer from the apex to the diagonal plane can be observed, although the model incluedes non-nearest neighbor interactions. In this poster, we propose a 3-dimensional model with only nearest-neighbor couplings, based on the 3-variable Krawtchouk polynomials considered in [2].

References

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PS04-2 (Orthogonal Polynomials, Mathematical Physics, and Interacting Particle Systems)

Orthogonal decomposition and mode expansion of gauge fields on S^2 in the presence of background gauge fields

Asai Kento[1], Yuki Honda[2], Joe Sato[2]

[1]Kyoto University, Kyoto, Japan
[2]Yokohama National University, Kanagawa, Japan

Email: honda-yuki-wy@ynu.jp

Higher dimensional gauge theories provide a promising framework for constructing theories beyond the Standard Model of particle physics. In particular, when a coset space is used as an extra dimension, its geometry naturally gives rise to background gauge fields. These background fields play an important role in determining the gauge symmetry and particle spectrum of the four-dimensional effective theory.

In our previous work, we performed mode expansions of gauge fields on the two-dimensional sphere S^2 in the presence of a background gauge field and analyzed the resulting background-dependent mass spectra. However, in that formulation, the separation between physical modes and Nambu–Goldstone modes was not fully clarified.

In this poster, we revisit the mode expansion by employing a Hodge-like decomposition based on covariant differential operators incorporating the background gauge field. By orthogonally decomposing the extra-dimensional components of gauge fields into exact, co-exact, and harmonic forms, we systematically separate physical modes from Nambu–Goldstone modes. We also discuss that this framework include the solutions obtained by the conventional Coset Space Dimensional Reduction (CSDR) method as harmonic modes. Furthermore, we present the Laplacian associated with this decomposition and discuss the corresponding orthogonal eigenfunction system.

PS04-3 (Orthogonal Polynomials, Mathematical Physics, and Interacting Particle Systems)

SWKB quantization condition and solvability via classical orthogonal polynomials

Yuta Nasuda

National Institute of Technology (KOSEN), Gunma College, Gunma, Japan

Email: y.nasuda@gunma-ct.ac.jp or y.nasuda.phys@gmail.com

Classical orthogonal polynomials (COPs) often appear in exact solutions of the Schrödinger equation for a certain class of potentials, known as conventional shape-invariant potentials (SIPs). The SWKB quantization condition is well studied in the literature, and is remarkable in that it reproduces exact bound-state spectra for all conventional SIPs, while it is never exact in other cases. This suggests a strong connection between the SWKB condition and COPs.

Recently, the author demonstrated that the SWKB condition equations for conventional SIPs are interrelated, in a manner analogous to the corresponding Schrödinger equations [1, 2]. These interrelations can be understood in terms of solvability via COPs. Moreover, the SWKB condition can be extended to other exactly solvable models associated with COPs [1]. In this poster, we present explicit examples and discuss the underlying connection between the SWKB quantization condition and COPs.

References

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PS07-1 (Quantum Walks and Quantum Information)

Testing k-block-positivity via semidefinite programs and its complexity

Qian Chen[1], Benoît Collins[2]

[1]The Chinese University of Hong Kong, Shenzhen, China
[2]Kyoto University, Kyoto, Japan

Email: chenqian.phys@gmail.com

We study the problem of testing k-block-positivity via symmetric N-extendibility by taking the tensor product with a k-dimensional maximally entangled state. We exploit the unitary symmetry of the maximally entangled state to reduce the size of the corresponding semidefinite program (SDP). We then analyze the complexity of the hierarchical SDPs, deriving an explicit formula for the complexity, which also clarifies why the SDP hierarchy collapses in the $k = d$ case. The presentation is based on arXiv:2505.22100 and arXiv:2601.19159.

PS07-2 (Quantum Walks and Quantum Information)

Quantum walks on the non-binary Johnson scheme

Kosei Nakamura, Hiroshi Miki

Doshisha University, Kyoto, Japan

Email: ctwn0915@mail4.doshisha.ac.jp

There are many examples of continuous-time quantum walks on distance-regular graphs, where interesting phenomena such as perfect state transfer (PST) and fractional revival are observed [1]. Particular examples are Hamming and Johnson graphs, which are associated with sasociation schemes, and it is reported that PST can be observed [2, 3]. In this poster, we focused on non-binary Johnson scheme, which is a generalization of Johnson scheme, and studied the quantum walks on the corresponding graphs.

References

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